Teacher's Guide

Part C:

Teaching Mathematics (P 4-6)

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Foreword

One of the biggest challenges Nigeria faces is how to ensure that the tuition provided in schools is of good quality such that pupils' learning outcomes improves significantly and those who complete primary school possess the requisite competences prescribed in the national curriculum. The current situation in which pupils' mean score in English, Mathematics, and Life Skills is only 30%-40% is a matter of concern to UBEC and all stakeholders.

To improve mean scores in the core subjects requires significant changes in the way teachers plan and deliver their lessons. It means building the capacity of teachers to make the transition from teacher centred methods to activity-based learner centred approaches. Teacher's Guide on Pedagogy, Literacy, Numeracy and Science & Technolgy has been developed by the Teacher Development Programme (TDP) seek to facilitate the adoption and use of active learning approaches in our classrooms.

UBEC is delighted to collaborate with TDP to make the Teacher's Guide available to schools in all parts of the country. Our expectation is that teachers will adapt and contextualise the Teacher's Guide to their local situation and use them to enhance the quality of teaching and learning in the classrooms. As soon as the Teacher's Guides are distributed to schools, teachers will be trained to use them as part of the UBEC-funded Teacher Professional Development programme.

I must thank DFID/UKAid and TDP for collaborating with us to improve the quality of teaching and learning in primary schools.

Dr Hamidu Bobboyi
Executive Secretary,
Universal Basic Education Commission,
Abuja.
6th March 2017

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Welcome and Introduction

Welcome to Teacher's Guide (Part C).

Teacher's Guide (Part A) provided study modules on general pedagogy in your classroom.

Teacher Guide (Part B) - Mathematics (P 1-3) included four study modules focusing on examples of teaching Mathematics for Primary 1 – 3 pupils, one each in the four key areas of

- Number and Numeration,
- Basic Number Operations leading to Algebraic Process,
- Measurement, and
- Geometry

This Part C of the Teacher's Guide contains study modules which provide an extension of these same areas but at a higher level appropriate for the P 4 – 6 curriculum. An additional Module has been included to introduce

- Everyday Statistics.

The Introduction to each study module indicates the year which is the main focus in each of the several sections. You should not ignore any section which is primarily focused on a year that you are not currently teaching. This is because, whatever class you have at the moment, you need to be aware of the progression for the topic. It is important that you know what learning is essential to precede your current work. It is important that you know the next steps to which your current teaching is leading. In this way you will have a greater understanding and mastery of the topic yourself. You will be a better teacher because you will be able to build on pupils' prior learning. Most importantly, you will be clear about the underpinning mathematical ideas that pupils need for them to gain mastery of the topic themselves. This knowledge will also enable you to differentiate appropriately, adapting your teaching to respond to the strengths and needs of all pupils.

Each module also includes activities and exercises for teachers, audio-visual clips, and questions for you to reflect upon. And, finally, each module contains suggestions for classroom activities. You are encouraged to study this material together with your partner in school so that you will both benefit from the professional insight that this will promote.

To get the most benefit from these modules, try out the different mathematical activities suggested in them and be adventurous in trying out new ideas. Some of the suggestions will help you to organise the learning for the very large classes that you sometimes face. Several of these ideas will be new to you but, whether concerning the teaching of Mathematics or advising you on classroom management, they are all suggestions of methodology which experienced teachers have found to be successful in helping pupils to learn.

During school support visits by the LGA Trainer or Teacher Facilitator, ask about the module contents and be prepared to discuss your experiences. Don't hesitate to question the ideas and to give feedback, both positive and critical, on all the materials.

All the best with the study!

Module 7: Number and Numeration

Module 7:
Number and numeration

As you know, Number and Numeration skills are the key components of Mathematics as an academic subject. But even for our pupils' daily interactions with their environment they need to be functional in basic number skills; and more so when they will be adults. Mathematician or not, we all need to be able to use numbers to perform the basic operations of addition, subtraction, division and multiplication as we deal with buying and selling, farming, working, building, travelling, and so on. In the Teacher Guide Part B, you have already studied some of these basic operations. In this Module these operations are developed further and extended to fractions, ratio and percentages. Children need to use all these types of numbers efficiently, accurately and reliably. This will help them to interpret data, to think logically and to reason carefully. Note that a significant part of number work needs to be done mentally.

This Module is designed to guide you on effective teaching about **Number**. Before you begin this number work, just spend a brief moment thinking about the reality of your classroom and recognise that a very significant part of pupil learning will be carried out mentally. Some, if not many, of your pupils will be without text books and exercise books in which they can write and practise mathematical calculations. You will often prepare posters and bring worksheets for pupils to write on and these will be important aids to assist learning. Nevertheless, your teaching will often be a conversation with pupils through which you will guide them and demonstrate new ways of working. You will be aware that clarity and simplicity will be key features of your teaching about mathematics. This clarity and reasoning does not mean rote learning. Do not expect pupils to just remember the steps in a calculation. Mental skills require deduction and reasoning. Allow pupils to understand why the steps are made and allow the pupils to have an ownership of the mathematical skills. Owning something means that you know it, can recognise it and can use it when you need to. Giving pupils an ownership and mastery of mathematical skills requires you to be explicit about the thinking processes that the pupils' mental skills will depend upon.

Consider, for example, teaching about prime numbers. It is of no value to expect children to remember and recognise prime numbers. The required skill depends upon pupils knowing that prime numbers only have 2 factors – the number 1 and the number itself. So the skill which pupils need to have is the ability to test any number for its factors. Consider, for example, 39. Is it at even number? No, so 2 is not a factor. Next, try 3; is 3 a factor?. Can you divide 39 by 3 ? Yes. So 39 has more than 2 factors: at least 1, 39 and 3 - possibly more than these three. But you don't need to test for any more factors because you already know it is not a prime number. Although it would be ideal if pupils can write in an exercise book and demonstrate their learning, this clear mental process of testing for factors gives pupils access to the mathematical understanding for identifying prime numbers. This relies upon their understanding and their thinking skills and not on rote learning. Their functional skill is to know how to test for prime numbers – not to try to memorise all the prime numbers. So it is the functional skill that you will need to teach, not the memorisation. Making the functional skill explicit enables learning to take place and ownership to be achieved.

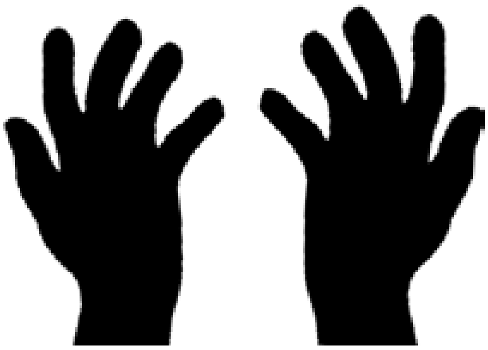
Objectives

By the end of this module, you will be able to guide pupils to be functional at

● calculating mentally with tens and hundreds	(Year 4)
● identifying proper and improper fractions	(Year 4 and Year 5)
● solving simple problems involving decimal fractions	(Year 4 and Year 5)
● using ratio to compare quantities	(Year 6)
● calculating with percentages	(Year 6)

Section 1:
Counting in bundles of 10s and 100s

Numeration (counting) is the basic concept of numbers which pupils learn to do from a very early age. The counting in bundles of 10s and 100s, which pupils began in Year 3, helps them to build an understanding of our base 10 (decimal) system for larger numbers.



People naturally count in 10s, probably because we have ten fingers.

Our fingers have always been used as a good aid for counting!

10 is written with a 1 and a 0 to indicate 1 pair of hands and 0 extra fingers.

14 is written with a 1 and a 4 to indicate 1 pair of hands and 4 extra fingers.

The number words, themselves, show their meaning; “fourteen” is a mix of the words “four” and “ten”.

It’s the same in Hausa 14 is *goma sha hudu* 19 is *goma sha tara* … and it’s the same in most languages, such as Fulfude where 14 is *sappo y nay.*

20 is 2 tens; 30 is 3 tens; 40 is 4 tens;…and so on…

Did you know that
the word *“digit”*,
used to name the figures in a
number, also means
“finger” or *“toe”* ?

Here, again, the number words like “forty”, “fifty”, “sixty”, … are a mix of the words “four tens”, “five tens”, “six tens”, and so on. It’s the same in older Hausa 30 is *gomiya uku*; 40 is *gomiya hudu*; 50 is *gomiya biyar* …and in Fulfude, where 30 is seppantati; 40 is seppannye. Many people now use the Arabic words *talatin* (30), *arba’in* (40), *hamsin* (50), and so on … These also literally mean “three tens”, “four tens”, “five tens”, … .

We write a zero after the single digit so that we can use the second number place for the number of 10s.

When we reach 10 tens, writing 100, we use three digits, writing 1 in the third place for the number of hundreds.

One hundred, two hundred, three hundred, …

“dari”, “dari biyu”, “dari uku”, …

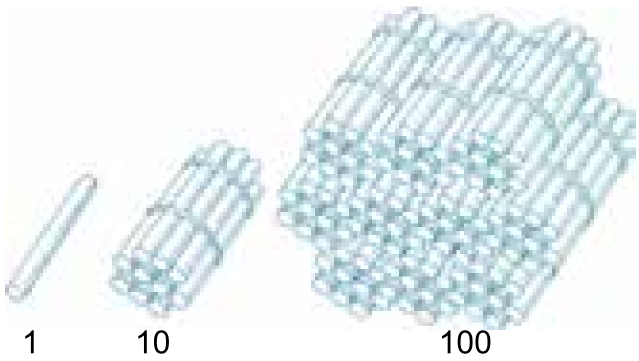
And, of course, when we reach 10 hundreds, we need a fourth place for thousands.

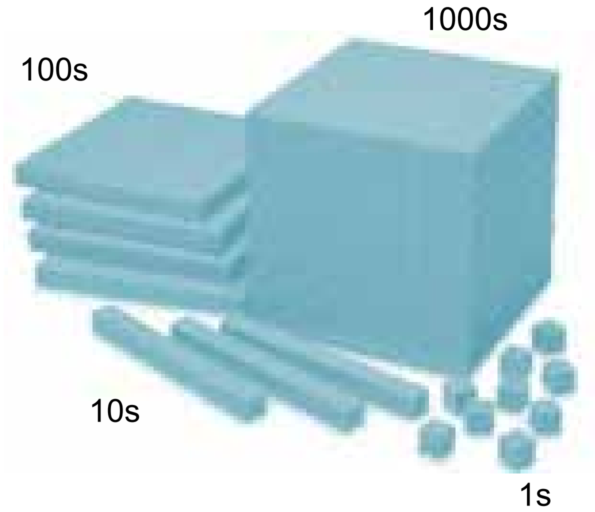
Then we count in 1000s. It’s the same in Hausa.

“dubu”, “dubu biyu”, “dubu uku”, …

To help children develop these number concepts, in the classroom we use bundles of thin sticks to model the counting. As a teacher of mathematics in primary school, you will need to make these bundles of 10 and 100. They are an essential teaching aid for P3 and P4.

It is not practicable to use bundles of 1000 sticks but some schools may use units, longs, flats and blocks to illustrate the U, T, H and Th of the base 10 number system.





Pupils will recognise that each bundle, or each block, contains 10 of the smaller unit and so each new object is 10 times bigger that the next smallest object. Counting up, and down, in tens, hundreds and thousands should be done from stated numbers, as well as from the bundles themselves:

SEVEN HUNDRED, EIGHT HUNDRED, NINE HUNDRED, ONE THOUSAND, ONE THOUSAND ONE HUNDRED,...

COUNT UP IN HUNDREDS, STARTING AT 536

FIVE HUNDRED AND THIRTY SIX, SIX HUNDRED AND THIRTY SIX, SEVEN HUNDRED AND THIRTY SIX, ...

COUNT DOWN IN HUNDREDS, STARTING AT 853

COUNT DOWN IN TENS, STARTING AT 119

COUNT UP IN THOUSANDS, STARTING AT 3602

Children will enjoy challenging and correcting each other with similar demands, in pairs or in their groups.

A note on number words

In Year 4, pupils will become fluent in the use of the English number words. The language structure for numbers is the same as in Hausa and in Arabic because they all share a common origin.

In fact, number words are often a useful guide to languages, their history and connections. This is because number words don't change their meaning over long periods of history. We can see that, as Islam spread slowly southwards across North Africa from the seventh century, Arabic became the language of cultural centres and Arabic numeration replaced or supplemented the indigenous systems. However, outside these centres, people lived their lives in the traditional way. This may explain why Hausa speakers traditionally count in base 10 and also use some Arabic words. For example, *shida* for 6 is an Arabic word which has replaced the old Hausa word and you may use the Arabic *alif* for 1000 instead of the old Hausa *dubu*.

Interestingly, old Hausa (like some older European languages such as English, French and Danish) has remnants of a much older base 20 system and you may hear *hauya uku* spoken for 60, meaning "three twenties". Some eastern Hausa speakers also use the subtraction principle with twenties for compound numbers ending in eight or nine; for example, 18 = 20 – 2; *(ashirin biyu gaira* or *ashirin biyu babu)* and 19 = 20 – 1 *(ashirin daya babu)* even though the Arabic *ashirin* has replaced the older Hausa words *hauya* or *laso.*

In Hausa, you may use *dari gaira biyu* for 98 (100 – 2) and *dari gaira daya* for 99 (100 – 1) and you may know 90 as *dari goma bus* or *dari gaira goma* (100 – 10) instead of *gomiya tara* or *tis'in*.

Similarly, *arbaminya gaira ashirin* is 380 using the old Hausa system 400 – 20 while, at the same time, the expression includes the more recent Arabic words for "four hundred" and "twenty".

Interesting, too, is the more recent use of *miliyan* and *biliyan* in Hausa for 1,000,000 and 1,000,000,000 showing the influence of the corresponding English words "million" and "billion".

Think

Think about your current classroom practice for teaching about counting:

1. Have you referred to the meaning of our number words?
2. Have you used thin sticks to make your own bundles of 10?
3. What else could you use to make bundles of 10 and 100?

Watch

Watch the video clip MM7V1.

You will see the teacher using bundles of 10 and bundles of 100 to remind the pupils of the work which they did to practise counting in Year 3.

In this video, you will only see the beginning of the lesson.

Think about the following questions as you watch the video clip.

1. How many bundles of 10 do you think the teacher needed for her lesson?
2. Why did the teacher repeat what had been done in Year 3 for her lesson starter?

Reflect

In the main part of this lesson, the teacher went on to use the bundles of 100s to support the children to give answers to questions such as:

500 + 200	600 – 200	2 x 300	800 ÷ 2

She used five bundles of 100 and added two more bundles of 100.

“How many sticks do I have now?”

Next, she placed six bundles of 100 and took two bundles away.

“How many sticks remain?”

Now she placed two groups of three bundles of 100.

“How many sticks do I have altogether in my 2 groups of 300?

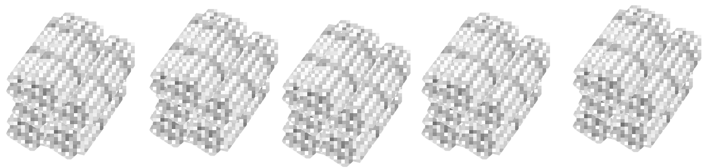
"Count with me to make a group of 800 sticks.

"100, … 200, … 300, … 400, … 500, …. 600, … 700, … 800

"Can we divide these 800 into 2 equal groups?"

For each mental calculation, the teacher also wrote the calculation on the chalkboard.

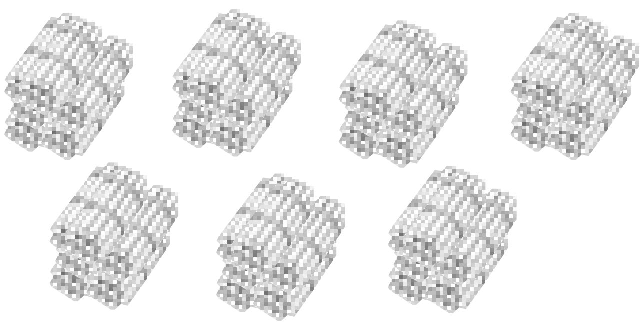
Then she asked children to solve some problems using the bundles of 100 sticks.



For the more able children, she asked:

"In these piles I have 500 sticks. Can I divide these into 2 equal groups?

"How many sticks will be in each group?"



"Can I give the group of 700 sticks to 2 people so that one person has 300 more than the other?"

The focus for the lesson was on pupils developing their skills to answer these questions mentally.

Why do you think that both the lesson starter and the main lesson were focussed on the practical use of the bundles of 10s and 100s to help pupils learn?

Work with your partner in the school

1. *What challenges might you encounter in your class organisation if you spent the whole lesson, starter and main part, on developing the pupils' mental skills?*

   How can you plan to overcome such challenges in teaching mental skills?

2. *What activities can help pupils to develop concepts of 10s. 100s and 1000s?*

   How big is a bag of 100 groundnuts? … 100 oranges?

   Could you hold 1000 groundnuts? Could 1000 oranges cover the floor of the classroom?

3. *How could you organise a similar lesson on children counting in 1000s? Bundles of 1000 sticks would be difficult to manage: could you use some 10, 100 and 1000 Naira banknotes to help develop pupils' mental skills for numbers in the thousands?*

To buy a mobile phone costing N 8,800 how many thousand Naira notes will you need, how many hundred Naira notes will you need? If you buy a phone charger (costing N 1,400) as well, how many thousand and how many hundred Naira notes will you need?

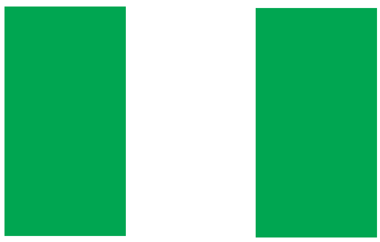
Check these Lesson Plans to help you link the counting activities with the notions of Place Value in writing numbers:

P4	Term 1	Week 1	Numbers	Day 1 Number 0 – 999
				Day 2 Revision of place value
				Day 3 Order numbers
				Day 4 Expand four-digit numbers
P4	Term 2	Week 11	Place Value	Day 1 Four-digit nu8mbers
				Day 2 Value of the digits
				Day 3 Playing with numbers
				Day 4 Finding numbers

Section 2:
Proper and improper fractions

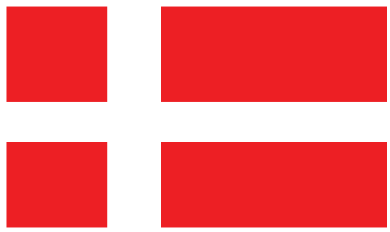
The concept of fractions is likely to be familiar to our pupils because they share things among themselves every day. At school they share their books, sweets and biscuits amongst friends. At home they share their food, their house chores, and so on.

The idea of sharing things is a good basis for pupils to understand the basic concept of fractions. You can build on this to teach about fractions and to calculate with them. The starting point will be to ensure pupils understand that, when we refer to fractions in mathematics, the shape or the quantity is divided equally.



Each part of the Nigerian flag is one third.

$^{1}/_{3}$ is white $^{2}/_{3}$ are green

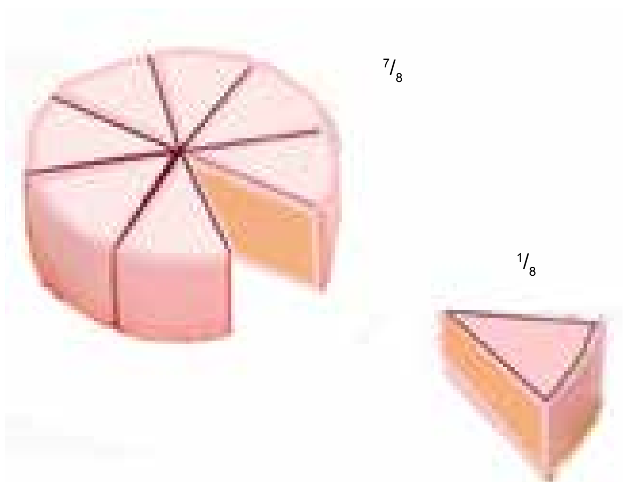


The five parts of the Danish flag are not fifths because they are not the same size.

The lesson plans for P4 will help you to teach fraction skills. Week 7 is all about fractions. The plans include examples for pupils to draw shapes cut into equal parts.

Pupils will learn how to represent fractions with diagrams. For example, this cake has been divided into eight equal parts.

Each piece is $^{1}/_{8}$. The result of dividing 1 by 8.



After Amira, Efome and Rabiu have each eaten their $^{1}/_{8}$ s, three eighths have gone, the remaining fraction is $^{5}/_{8}$. ***How can you illustrate*** $^{5}/_{8}$ ***?***

Children can find fractions difficult because the two numbers which we use to write them are each doing a different job.

In $^{5}/_{8}$, the "5" tells us ***how many pieces*** remain and the "8" tells us ***the size of the pieces***.

Because the numbers in a fraction do different jobs, they have different names – the numerator (the one on top which tells you how many pieces) and the denominator (the number on the bottom which tells you into how many equal parts the whole object has been cut).

Is $^{1}/_{10}$ smaller or larger than $^{1}/_{8}$?

HOW DO YOU KNOW?

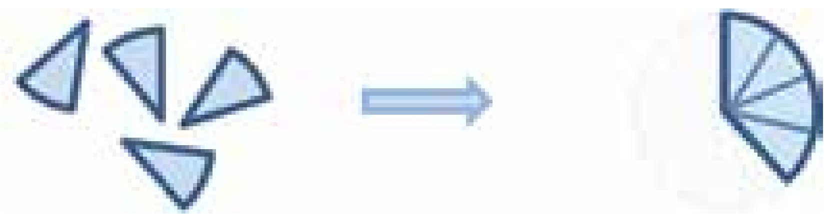
Which is larger: $^{1}/_{2}$ or $^{1}/_{3}$?

HOW WILL YOU HELP CHILDREN TO KNOW THAT THE FRACTION WITH THE LARGER DENOMINATOR HAS THE SMALLER PIECES?

If the Danish flag is cut into 35 small equal pieces, you can see the fraction of the flag that is red is $^{24}/_{35}$.

WHAT FRACTION OF THE DANISH FLAG IS WHITE?

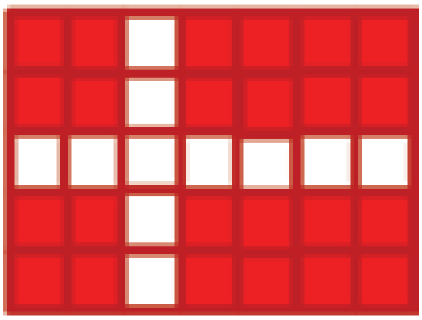
In this section we will be looking at the names given to fractions.
Here are **four elevenths**:



$^{4}/_{11}$ $^{4}/_{11}$

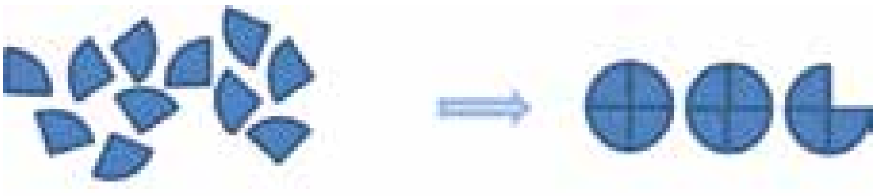
Here they are drawn as separate pieces. Each piece is $^{1}/_{11}$

Here they are drawn as part of a whole.



A **proper fraction**, like $^{4}/_{11}$, is part of one whole. It is smaller than one whole because the number of pieces (the numerator 4) is smaller than the denominator 11 (the number that indicates the size of the pieces).

Now look at **eleven quarters** (some people say "eleven fourths"):



$^{11}/_{4}$ $2^{3}/_{4}$

The eleven pieces are each $^{1}/_{4}$

$^{11}/_{4}$ is an improper fraction, not because is wrong, but because this number would normally be written as the mixed number $2^{3}/_{4}$. (Four quarters are enough to make a whole one, so eleven quarters can make two whole ones and there are three quarters more: two and three-quarters.)

An improper fraction can easily be recognised because the number on the top is more than the number on the bottom. It can always be simplified to a mixed number. The simplification of improper fractions will be taught more fully in Year 5.

4/4 is also an improper fraction. Again, this is not because it is wrong, but because this number would normally be written as 1. *Can you explain why?*

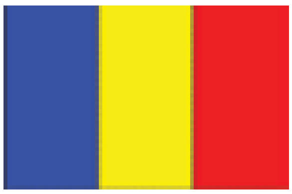
Your answer to this question is an appropriate and an important idea for Year 4 pupils because it emphasises that 4 quarters make a whole … that 10 tenths make a whole and *(another example)* 12 twelfths make a whole. *How many fifths make a whole one?*

It may seem puzzling to talk about improper fractions but, as children will find in Year 5 and Year 6, these ways of writing mixed numbers can often make calculations with fractions easier to manage.

These fraction names are not, in themselves, important. It is more important that children in Year 4 —

- can recognise how many fraction pieces make a whole;
- know that each fraction part is the same size; and
- have the ability to rewrite improper fractions between 1 and 2 as mixed numbers.

Describing the fractions on flags makes an interesting lesson in which pupils will learn these ideas. Pupils can design their own flags and describe them using fractions. The individual sections of each flag must be the same size but several sections can be the same colour, like the Latvian or the Danish flags.



Gabon

Chad

Latvia

On the flag of Latvia, the brown stripes are twice as wide as the white stripe and so the extra lines placed on this picture show that the brown stripes are each $^{2}/_{5}$ of the flag.

The total of the flag which is brown is $^{4}/_{5}$ because $^{2}/_{5} + ^{2}/_{5} = ^{4}/_{5}$

Pupils will need to recognise that only the 2s are added in this calculation because the 5s (the denominators) are only indicating the size of the fraction pieces.

Your answer to the question about the fraction of the Danish flag which is white was $^{11}/_{35}$ because $^{11}/_{35} + ^{24}/_{35} = ^{35}/_{35}$ (which is 1 whole). Pupils will need to learn that $^{11}/_{35} + ^{26}/_{35}$ does not equal $^{35}/_{70}$ because the denominators 35 only describe the size of the small squares as parts of the rectangle. The denominator tells you how many small squares the whole flag has been divided into to describe the fraction pieces. The denominators are not added because they are only labels.

You may have answered the Danish flag question by thinking that the whole flag has 35 small squares and so, to calculate $1 — ^{24}/_{35}$ you may have subtracted $^{35}/_{35} — ^{24}/_{35} = ^{11}/_{35}$

You would not have written this calculation as $^{35}/_{35} — ^{24}/_{35} = ^{11}/_{0}$ because this wouldn't make sense. Of course, pupils are likely just to count the white squares in the Danish flag and tell you there are "11 thirty-fifths" that are white but the example is a good way to show how fractions are added together, or subtracted, without changing the denominators.

Similarly, the Nigerian flag is $^{2}/_{3}$ green because $^{1}/_{3} + ^{1}/_{3} = ^{2}/_{3}$, not $^{2}/_{6}$ Pupils will need to learn that fractions can only be added together if they are made of the same size pieces.

They can add $^{3}/_{10} + ^{5}/_{10} = ^{8}/_{10}$ and subtract $^{4}/_{5}$ -- $^{1}/_{5} = ^{3}/_{5}$ because the “fraction” pieces in each calculation are the same size.

Year 4 pupils will know that two quarters are the same quantity as one half. So they can add $^{1}/_{2} + ^{1}/_{4}$ (although these are different size fractions) by changing the addition to $^{2}/_{4} + ^{1}/_{4} = ^{3}/_{4}$

This will emphasise the need for fractions to have the same denominators in order to add or subtract the fractions.

In Year 6, pupils will learn how to add, or subtract, fractions of different sizes by changing them to equivalent fractions which have the same denominators, as we did here with the half and the quarter. But those calculations will first require more work on recognising which fractions are the same size as other fractions, such as the Year 5 examples given in the Summary section of this Module.

Think

- Have you ever taught the understanding of a unit fraction being an **equal** part of a whole?
- What did you use to help children understand that a fraction is part of a whole?
- How can you link the topic of fractions with real life examples?

Watch

Watch the video MM7V2.

1. How did the teacher encourage the pupils’ participation in the lesson?
2. Write an example of an improper and a proper fraction seen in the video.
3. What materials did the teacher use in the lesson?

Reflect

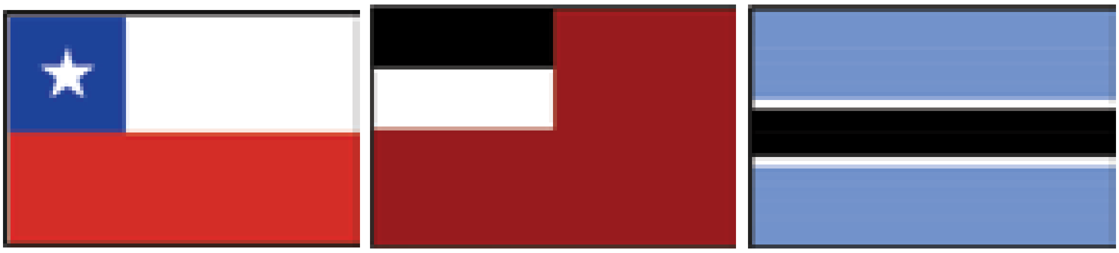
- Do you agree that "explaining by using pictures is a more effective way to teach fractions?

Write three reasons.

- How would you recommend this teacher to help children to know…

  …how many fractions of a given size are needed to make one whole?

  …how to change an improper fraction like $^{5}/_{4}$ to a mixed number?



Chile Republic of Georgia Botswana

Work with your partner in school

1. Discuss with your partner how you could use flags like these to help pupils understand ***equivalent fractions*** in Year 5. You will need to add some extra lines to these flags so that pupils can begin to understand that some fractions are equivalent in size to other fractions.

2. Discuss with your partner how you will teach a lesson that helps pupils to change an improper fraction like $^{3}/_{2}$ or $^{5}/_{4}$ or $^{11}/_{10}$ to its equivalent mixed number.

   Will your lesson include changing an improper fraction like $^{10}/_{10}$?

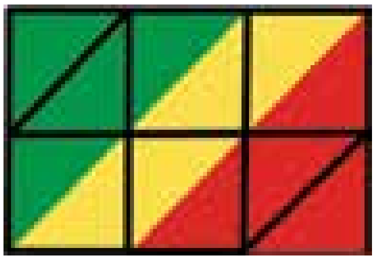
   Ask your partner to watch you teach your lesson and to give you some feedback.

In describing the Chile flag, pupils will realise that $^{3}/_{6}$ is equivalent to $^{1}/_{2}$.
"Three out of six is the same as a half."

The Georgia flag demonstrates that 68 is equivalent to $^{3}/_{4}$. *"Two eighths are the same as one quarter."*

The Botswana flag has two blue bands that are each $^{10}/_{30}$ of the design so the flag is $^{2}/_{3}$ blue.

The middle third is $^{4}/_{30}$ white (the white stripes are $^{2}/_{30}$ each) and $^{6}/_{30}$ black.



The Republic of Congo has an interesting flag for looking at fractions. Pupils will need to draw in some extra lines to recognise that the three colours are each one third of the design.

- *How many small triangles make up this design?*
- *How many of them are green?*
- *How does the flag show that $^{4}/_{12}$ are the same as $^{1}/_{3}$?*

You may decide that these interesting flag questions are more suitable for Year 6 pupils. You are the best person to judge whether they will be appropriate for your class or, perhaps, for the more able pupils in a Year 4 group.

As mentioned above, the Lesson Plans provide more examples to help you teach about fractions.

See, especially, P4 Week 7 *Fractions*	Day 1 Making fraction strips
	Day 2 Understanding numerator and denominator
	Day 3 Ordering fractions
P5 Week 22 *Fractions*	
P6 Week 6 *Fractions*	
Week 22 *More Fractions.*	

Section 3:
Decimal fractions

Pupils use fractions on a daily basis without thinking about it. *Half a cup of water; half a bottle of red oil; a quarter cup of garri* … So children tend to have a good understanding of the common everyday fractions. As we saw in Section 2, Year 4 pupils will easily recognise that

$^1/_2$ plus $^1/_4$ will equal $^3/_4$ when they realise that $^1/_2$ is equivalent to $^2/_4$

It is unlikely that children will be familiar with decimal fractions and so these need to be introduced with caution. As you know, our counting systems, both ancient Hausa and modern world numbers, are based upon the system of counting in 10s – the decimal system ("deca" being the ancient Indo-Aryan word for "ten"). The **decimal fraction** notation is the extension of that system to fractions and so is based upon $^1/_{10}$, $^1/_{100}$, $^1/_{1000}$, and so on.

The first step with decimal fractions is for children to know that they are just an alternative way of writing the fractions they already know.

Instead of writing $^3/_{10} + {}^4/_{10} = {}^7/_{10}$ they will use the new notation 0.3 + 0.4 = 0.7, because 0.1 is the decimal way of writing $^1/_{10}$. That is what it means.

The difficulty for pupils will be that, although a fraction such as $^1/_2$ can be changed to the equivalent $^5/_{10}$ to write it as 0.5 , other fractions such as $^1/_4$ cannot be expressed in tenths.

To add one quarter in decimals, we will need to use hundredths so that we translate $^1/_2 + {}^1/_4 = {}^3/_4$ to $^{50}/_{100} + {}^{25}/_{100} = {}^{75}/_{100}$ which, in the new notation, is 0.50 + 0.25 = 0.75

Note, here, that we say "*zero point five zero"* plus *"zero point two five*" equals "*zero point seven five*". Pupils need to learn that we do not say "zero point seventy-five". We say each decimal fraction place separately so that the decimal fractions are not confused with whole numbers.

Well, that is getting ahead rather too quickly, because to change a fraction such as $^1/_4$ to $^{25}/_{100}$ and $^3/_5$ to $^6/_{10}$ or to $^{60}/_{100}$ requires work which will come later. So first let us consider what is needed in Year 4.

From the Section 2 work on Fractions, children will know how to count in tenths; they will know that $^1/_{10} + {}^6/_{10} = {}^7/_{10}$; they will have learnt that $^7/_{10} + {}^4/_{10} = {}^{11}/_{10}$ and they will know to write this as $1^1/_{10}$;

Now they will learn to write these tenths using the decimal notation.
0.7 + 0.4 = 1.1

At first, it will be a translation exercise. When they are comfortable with translating their tenths into decimals, they will no longer need to translate; they will be able to add numbers with one decimal place directly. This work will be supported by measurements using centimetres and millimetres.

Because pupils will know that 10mm are equal to 1cm, they will know that 1mm is $^{1}/_{10}$cm, but tell pupils to use 1 mm = 0.1cm rather than 1/10cm.

To support this development of pupils' understanding of decimals, provide pupils with some everyday examples of measurements in which decimals are used.

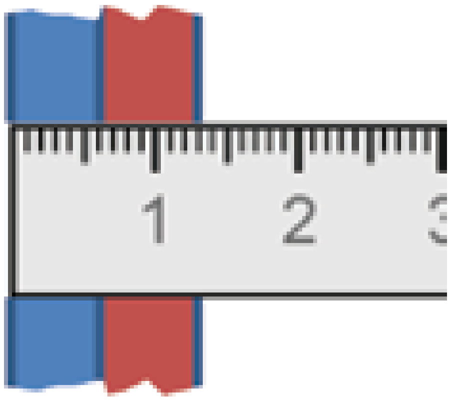
The diagram below shows two boards placed together

The blue board is 6mm thick. That's 0.6cm. The brown board is 7mm thick. That's 0.7cm. Pupils should become familiar with describing the thickness of these two boards in several different ways – all of which are equivalent.

6mm + 7mm = 13mm

$^{6}/_{10}cm + {}^{7}/_{10}cm = {}^{13}/_{10}cm = 1{}^{3}/_{10}cm$

0.6cm + 0.7cm = 1.3cm

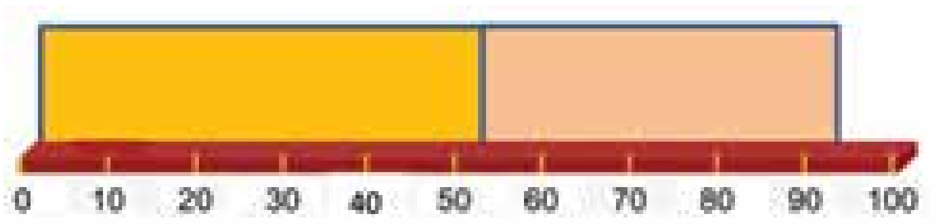


From the Section 2 work on Fractions, pupils will also know how to count in hundredths;

They will know that $^{23}/_{100} + {}^{36}/_{100} = {}^{59}/_{100}$; and they will have learnt that $^{75}/_{100} + {}^{25}/_{100} = {}^{100}/_{100}$ and they will know to write this as 1.

Now they will learn to write these hundredths using the decimal notation.

0.23 + 0.36 = 0.59; it will be another translation exercise.



When they are comfortable with translating their hundredths into decimals, they will no longer need to translate; they will be able to add numbers with two decimal places. This work will also be supported by measurements.

Because pupils will know that 100cm are equal to 1m, they will know that 1cm is $^{1}/_{100}$ m

The yellow board is 52cm long. The buff coloured board is 42cm long. The metre ruler shows that their total length is 94cm.

The two boards are 6cm shorter than 1m. That's 0.06m because they are $^{6}/_{100}$ m less than a metre. What would be the total length of three yellow boards fixed length to length?

Can you answer this question using

1. cm ?
2. fractions of a metre ?
3. decimal fractions of a metre ?

These calculations all give the answer to the question of the length of the three yellow boards:

52cm		0.52m	
52cm	52cm	0.52m	0.52cm
+ 52cm	x 3	+ 0.52m	x 3
156 cm	156cm	1.56m	1.56cm

$^{52}/_{100}$ m + $^{52}/_{100}$ m + $^{52}/_{100}$ m = $^{156}/_{100}$ m = $^{156}/_{100}$ m

Think

Thinking about your current classroom practices, answer the following questions:

1. What methods and materials have you been using to teach decimal fractions?
2. What difficulties have you encountered when introducing decimal fractions?
3. What was the level of pupil involvement in the lessons?

Watch

Watch the video clip MM7V3.

1. How did the teacher encourage pupils' participation in her teaching of decimal fractions?

2. What assessment techniques did the teacher use in the lesson?

Reflect

- You may have noticed that, when doing the group work, one group of boys in the lesson had written

$$\begin{array}{r} 0.7 \\ 0.9 \\ \hline 0.16 \\ \hline \end{array}$$

and that this had been crossed out and replaced by

$$\begin{array}{r} 0.7 \\ 0.9 \\ \hline 10.6 \\ \hline \end{array}$$

How might the teacher have helped this group to understand their errors?

- Imagine you are a mentor of a P4 teacher who is going to introduce the concept of decimal fractions to his pupils tomorrow. He has studied the lesson plan already but feels nervous as he has never taught this topic before. What three suggestions will you give him regarding teaching decimal fractions… and why?

	Suggestion	Why?
1		
2		
3		

Work with your partner in the school

Here are three points to discuss with your partner

1. What challenges might you encounter in your class organisation if you applied the method in the video to teach decimal fractions?

   How can you plan to overcome such challenges in teaching decimal fractions?

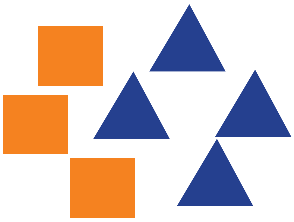
2. How might you explain that 10/100 is the same as 0.1 ?

3. Why is it important to demonstrate that $^4/_{10} + ^5/_{10} = ^9/_{10}$ before teaching that 0.4 + 0.5 = 0.9 ?

Section 4:
Ratio

We use ratio to make comparisons between two quantities.
Ratio is a way of comparing amounts of something – how much of one thing there is compared to how much of another thing.
When we express a ratio in words, we use the word “to”.
When we write a ratio in mathematics, we use the colon symbol “ : ”

Let us look at some examples. First, compare the numbers of different objects.



In this group of shapes, the ratio of squares to triangles is three to four.

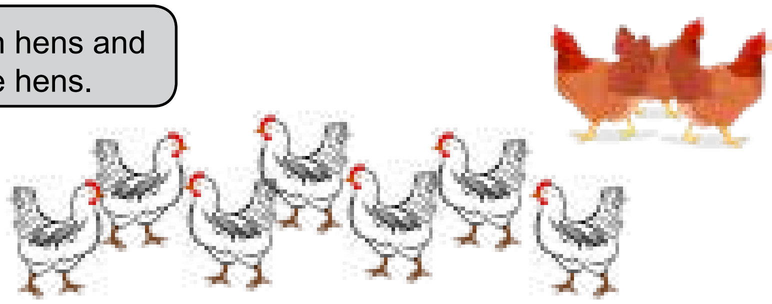
We write “The ratio of squares to triangles is 3 : 4 ”

We could also write “The ratio of triangles to squares is 4 : 3 ”

We simply need to make sure that the order of the numbers matches the words used.

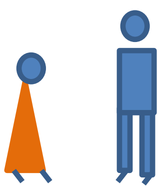
Now, compare different amounts of the same objects.

Musa has 3 brown hens and Amina has 7 white hens.



“The ratio of Amina’s hens to Musa’s hens is 7 : 3 ”
Ratio sounds easy. You just compare the numbers in each group.

Often, the numbers in a ratio enable you to know how many times bigger one group is, compared to another. That’s what makes ratio useful, mathematically. But that’s what makes ratio more difficult for children to handle and this is the reason why we don’t teach ratios until Year 6.



Amina is 10 years old and her teacher is 30 years old.

The teacher is 3 times as old as Amina.

“The ratio of the teacher’s age to Amina’s age is 30 : 10 ”

We can simplify this ratio of 30 : 10 to the ratio 3 : 1

What is the ratio of pineapples to mangoes in this picture?
You probably said 3 : 6
You could also have said 1 : 2
Both answers are correct but the second answer has been simplified.
It recognises that when we compare these fruits, there are 2 mangoes for every 1 pineapple.

When thinking about ratios, children can easily be confused by the difference between the two numbers compared. In this example of the fruit, they will see that the mangoes are three more than the pineapples.

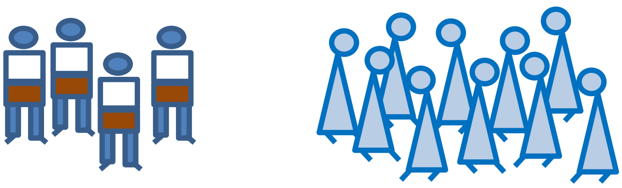
But ratio is not concerned with the difference; it is concerned with the relationship between the two groups. How much bigger is one group that the other? Ratio describes a multiplicative relationship. Hence, to simplify a ratio, recognising multiplication factors is important. The ratio 2 : 1 shows that there are twice as many mangoes as pineapples.

When we compared the ages of Amina and her teacher, we recognised that their ages were 1 x 10 and 3 x 10. So we could simplify the ratio to 1 : 3

The factor 10 does not appear in the ratio itself but it is the important factor which enables us to simplify the ratio. The ratio 1: 3 shows that the teacher is three times older than Amina.

If there are no common multiplicative factors, like in the first example of squares and triangles where the ratio was 3 : 4 , then the actual numbers are the simplest way to compare the quantities.

Note that we only use whole numbers to simplify a ratio.



In Amina's family, the ratio of boys to girls is 4 : 10

We could simplify this to 2 : 5 because there are 5 girls for every 2 boys.

But we would not write 1 : 2½ even though we recognise that the ratio 2 : 5 shows there are 2½ times more girls than boys.

Which one of the following statements does not describe a ratio?

A. Hussain has five times more goats than cows.

B. Hussain has sixteen more goats than cows.

C. Hussain has 5 goats for every cow.

D. The ratio of Hussain's cows to his goats is: 4 : 20

Hussain has 20 goats and 4 cows. A ratio of 5:1
The second statement (B) is true but it's not a ratio because it doesn't say how many times bigger the number of goats is. It just describes the difference between the numbers of animals.

Think

Thinking about your current classroom practices,

7. What are the common mistakes that pupils make with ratio?
8. "Chalk and blackboard are enough to teach ratio because it's just about numbers." Do you agree or disagree that no teaching aids are needed when introducing ratio?

Watch

Watch the video clip MM7V4.

5. Were the learning materials appropriate to teach ratio?
6. What assessment techniques did the teacher use in the lesson?
7. What learning took place? Look for evidence of this in the video clip and then discuss this with your partner in school.

Reflect

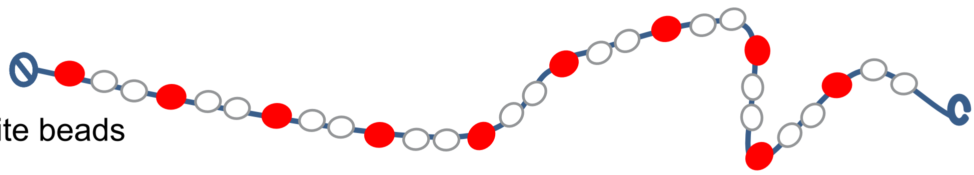
1. You have three children with different needs in your class:
   - A child who is a slow learner
   - A child who is a visual learner
   - A child with hearing difficulties

How would you help each of them to understand ratio more clearly?

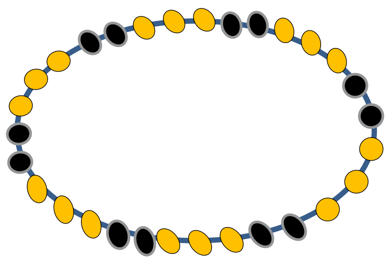
2. In the video, the teacher asked the children to compare 4 pencils with 6 books. The quantities allowed him to introduce the ratio 4 : 6
   How might he use the same example in the next lesson to show that the ratio 4 : 6 is the same as the ratio 2 : 3 ?

Work with your partner in the school

Discuss with your partner how this necklace and bracelet could help you teach children about ratio.



red beads : white beads
10 : 20
1 : 2



gold beads : black beads
18 : 12
3 : 2

- Can you find a suitable bracelet, necklace or waistband that could help you to teach ratio? Perhaps you could make some like these, using beads or seeds?
- Children will like to make their own necklaces or bracelets using dried seeds. Such an activity can provide a useful link between Mathematics and Nature Study as well as providing for a conceptual understanding of ratio.

Section 5
Percentages

Having introduced Year 4 pupils to *Fractions* and then to *Decimal Fractions,* they will have spent some time in Year 5 developing their ability to calculate with these fractions. But, because proper fractions and decimal fractions can be difficult for people to work with, in Year 6 we introduce a third type of fraction called *Percentages*. The word **percentage** comes from "per cent" which means "per hundred".

The idea behind using percentages is to express every fraction as part of 100 – the intention is to make any fractions easy to compare. You will know that comparing $^2/_5$ of a quantity with $^3/_8$ is difficult because fifths and eighths are different size fractions and unless we change them both to fortieths ($^{16}/_{40}$ and $^{15}/_{40}$ respectively), we would not know which was the bigger fraction.

Instead of working with fractions with different denominators and instead of working with decimal fractions using tenths, hundredths and thousandths, using percentages enables all quantities to be described as hundredths.

$^1/_4$ translates to $^{25}/_{100}$ 0.4 is $^{40}/_{100}$ $^4/_5$ translates to $^{80}/_{100}$ $4^1/_4$ can be written as $^{425}/_{100}$

which enables these to be written, with the shorthand symbol for percentages, as

25% 40% 80% 425%

Because these quantities are all expressed as their equivalent part of 100, they can easily be compared and used in calculations.

Learning about percentages requires the mastery of three skills:

- Firstly, the ability to use a percentage to represent any fraction as part of 100.
  For example, to know that 75% is equivalent to $^3/_4$ because 75% means $^{75}/_{100}$
- Secondly, to find a percentage of a quantity.
  For example to be able to calculate 75% of ₦2,000 and know that this is ₦1,500
- Thirdly, to express one quantity as a percentage of another.
  For example, to calculate that 45 boys in a class of 60 pupils is 75% of the class.

Your first lessons will be to teach pupils to translate (and to simplify, where possible) percentages to the fractions they represent.

50% 10% 150% 20% 12.5% $33^1/_3$% $66^2/_3$% 62.5%

As you see here, you will sometimes meet fractions and decimals within a percentage! Conversely, pupils will learn how to change fractions and decimals into their equivalent percentages.

$^1/_3$ $^1/_5$ $^1/_8$ 0.1 $^2/_3$ 0.5 $^5/_8$ $1^1/_2$

Check that you can match the brown percentages with the grey fractions.

This work will have been made much easier if, in Year 5, pupils have already found fractions of 100 as part of their work with proper fractions.

Calculate one fifth of 100

What is $^3/_5$ of 100?

Pupils may find it strange to use 150% to represent $1^1/_2$ but they will probably be familiar with the idea of making 100% effort on a task. Now you will help them to know that, for example, 200% means "twice as much".

Later lessons will be concerned with finding percentages of quantities.

50% of ₦2,000

25% of 40 cows

12.5% of 80 pupils

10% of 20 litres of water

150% of 10 hectares

20% profit on ₦10,000

These calculations will sometimes be done by treating the percentages as simple fractions, particularly if the quantities are convenient multiples of the denominator. For example, to find 12.5% of 80 pupils, recognising that 12.5% is equivalent to $^1/_8$ makes this an easy calculation. Pupils will develop their skills through a progression of activities such as the following:

Pupils explore, using their knowledge of 100% to find other quantities.

100% = ₦240

50% =

25% =

1% =

10% =

5% =

1% =

20% =

"What other percentages can they find?"

Then, pupils find a given percentage by choosing only the combinations which they need to produce the required amount:

100% = ₦700

50% =

10% =

5% =

I FOUND 65% BY CALCULATING 50% AND 10% AND 5%.

The process can then be reversed to facilitate problem-solving:

The price of the phone was reduced by 20% to ₦6,400

What was its original price?

80% = ₦64,000

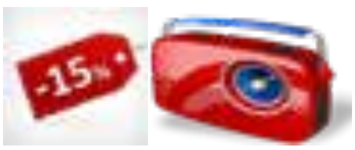
1% =

10% =

100% =

The next important stage in the development of the process skill is the recognition that, whatever percentage of a quantity you are given, you can always find 1% of the quantity.

The price of this radio has been reduced to ₦8,500



"How can you find 1%?"
"If you can find 1%, how can you find 15%?"
"What price would 100% have been?"

Finally, pupils will learn how to express one quantity as a percentage of another. This will involve changing the proportion to an equivalent fraction of 100. Here are three examples:

A football team wins 10 out of the15 games that it plays.
That's $^{10}/_{15}$ of the games played. $^{10}/_{15}$ out of 100 would be $66^{2}/_{3}$
So they have won $66^{2}/_{3}$% of their games.

Salim sores 148 out of 200.
His mark is 74% because
$^{148}/_{200}$ is equivalent to $^{74}/_{100}$

Hassana scored 64 out of 80.
That's equivalent to 8 out of 10 so, as a percentage, her mark is 80%.

Each calculation has been done differently but, in each case, the quantities 10 out of 15; 148 out of 200 and 64 out of 80 have been changed to an equivalent proportion out of 100 to express it as a percentage.

Think

1. If 40% of your class are girls, what percentage are boys?
2. When petrol prices increased by 25% because of fuel shortages in your area last month, by what percentage must the new price decrease to go back to the old price?

Watch

Watch the video clip MM7V5. As you watch the three parts of this revision lesson, think about

1. how the teacher encouraged pupils to participate;
2. the different teaching aids shown in the clip;
3. which question on percentages was the question that most pupils responded to.

Reflect

1. How easy do you think it is to make the instructional materials shown in the video clip?
2. Did you observe any change in the pupils' behaviour as the teacher praised their efforts?
3. What other methods could the teacher use to evaluate the pupils' performance?

Work with your partner in the school

The teacher in the video checks that pupils know what ***percentage*** means. He confirms that a percentage describes a fraction of 100: that 8% of a quantity means $^{8}/_{100}$ of the quantity. The teacher reminds the class that percentages can be illustrated as a fraction of 100 square and they build up a diagram of 100% in four parts.
He then reminds pupils how to calculate a 10% discount on an item for sale and the pupils demonstrate that they can calculate such discounts for a variety of items.
Discuss with your partner

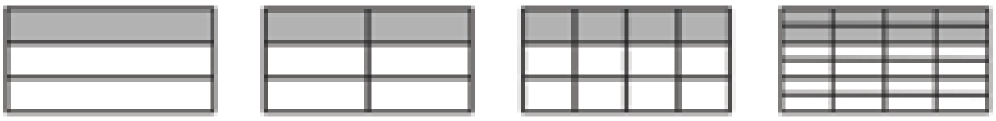
- *which percentage skills are not included in this teacher's revision lesson;*
- *a sequence of five lessons for pupils who are to be introduced to calculating with percentages.*

Summary of Module 7

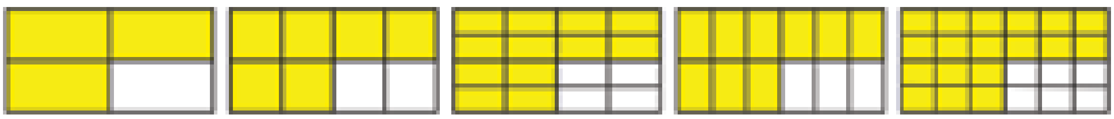
Number and numeration are the key features of all aspects of learning mathematics. Pupils also use basic numbers in their daily lives, at home, at school and in the market. The money we use has numbers so that we can determine the different values that each coin or note represents. We use numbers for knowing the time, for the measurements of clothes we need, and for measuring all the sorts of the quantities and shapes that we use in a modern society.
Sadly, many people find mathematics difficult to handle, especially when confronted with fractions, decimals and percentages. However, research shows that that many people find difficulty with numbers (and hence have a dislike of mathematics) simply because of the way in which they were taught at school. When teachers use relevant and interesting ideas, with a variety of no-cost or low-cost materials, pupils find number work much more accessible.
As a teacher, you have a significant responsibility in helping pupils to succeed and to enjoy mathematics: if you use only the rote learning procedures which have caused people to dislike mathematics, you will not succeed in helping pupils to master mathematical ideas and to have an ownership of the numerical skills which are the basis of mathematics. Here are some ideas to help you to teach number work effectively.

Ideas to try in the classroom

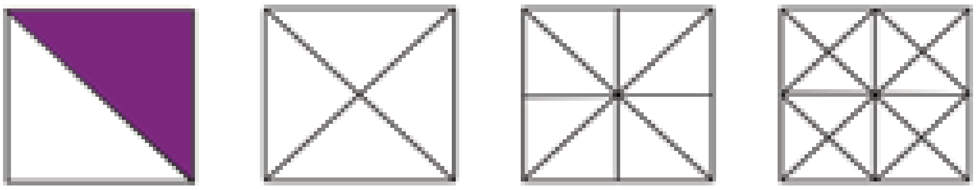
Equivalent Fractions



These diagrams show that $^1/_3$, $^2/_6$, $^4/_{12}$, and $^8/_{24}$ are equivalent.



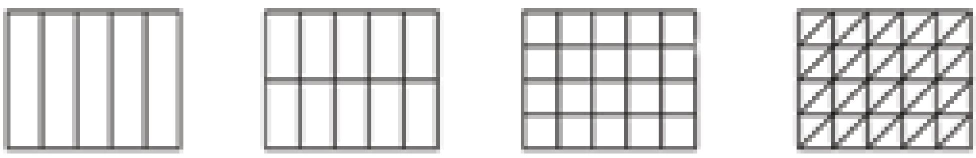
1. What do these diagrams show?



2. Copy and shade these diagrams to show that $^1/_2$, $^2/_4$, $^4/_8$, and $^8/_{16}$ are equivalent fractions.

Copy the three sets of diagrams below.
Use them to show three more sets of equivalent fractions.

3.



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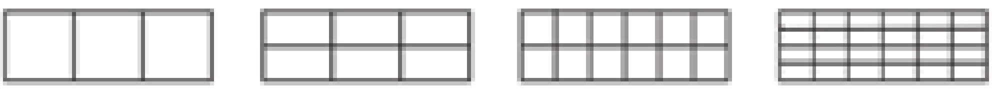

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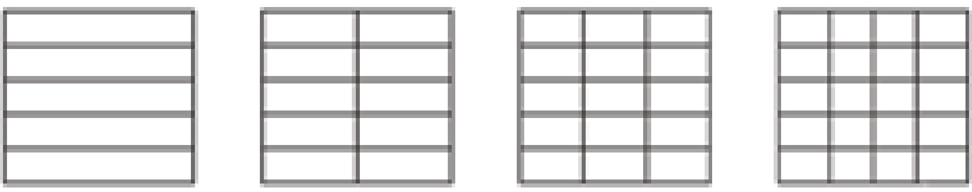
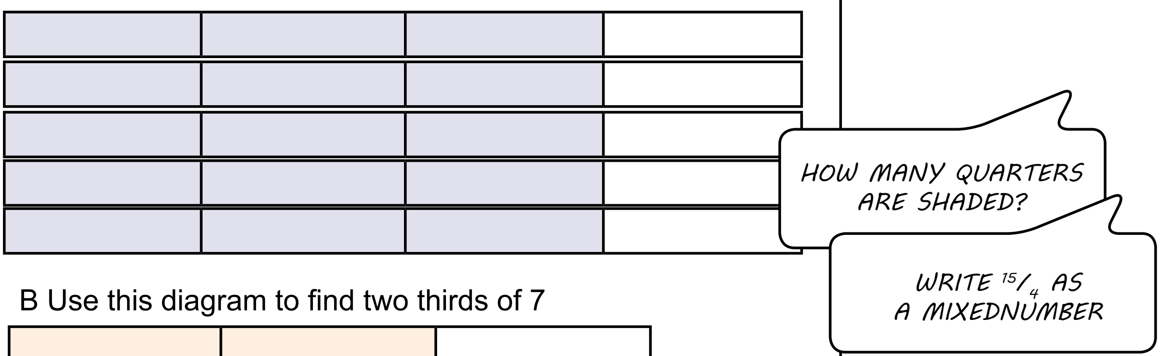


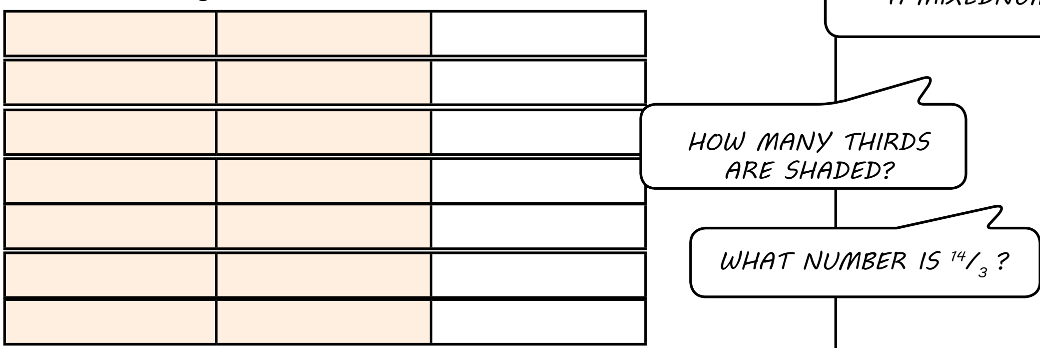
When pupils are able to shade fractions of a rectangle, ask them to explore what answer they can give for the number which is $^{3}/_{4}$ of 5 … and $^{2}/_{3}$ of 7

Explore fractions of a quantity

A. Use this diagram to find what number is three quarters of 5



B Use this diagram to find two thirds of 7



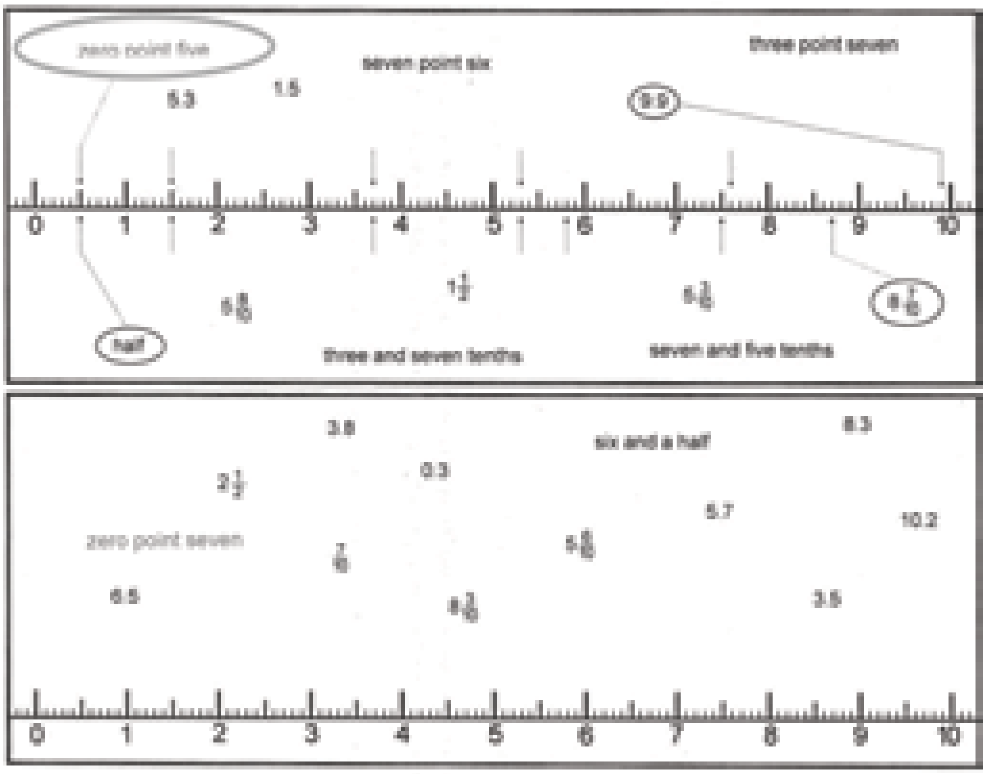
Can you extend this activity to find the number which is $^{3}/_{4}$ of 500 … and $^{2}/_{3}$ of 700 ?

To introduce decimal fractions, pupils can match fractions to positions on a number line. Use proper fractions, mixed numbers and improper fraction formats. Use the number words too.

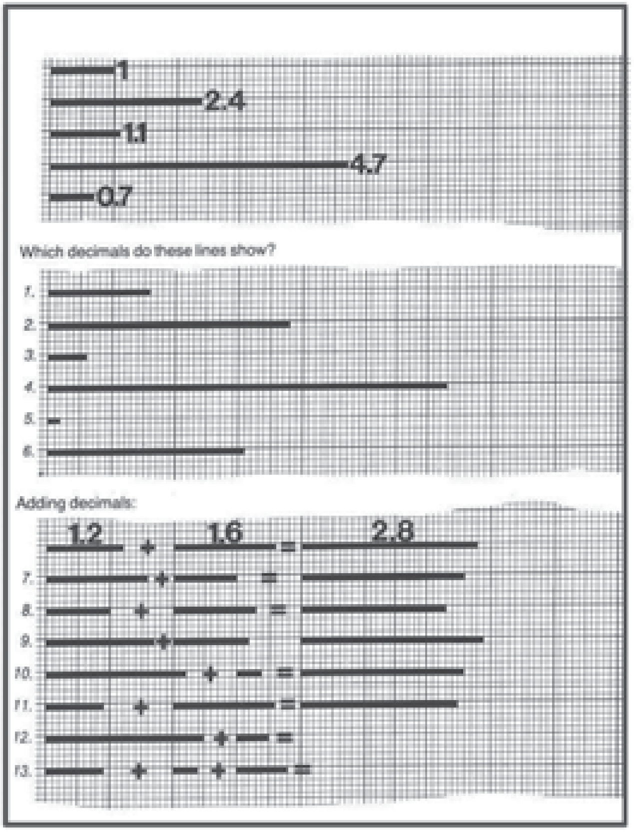
1.
Match the numbers and words to the arrows on the number line.

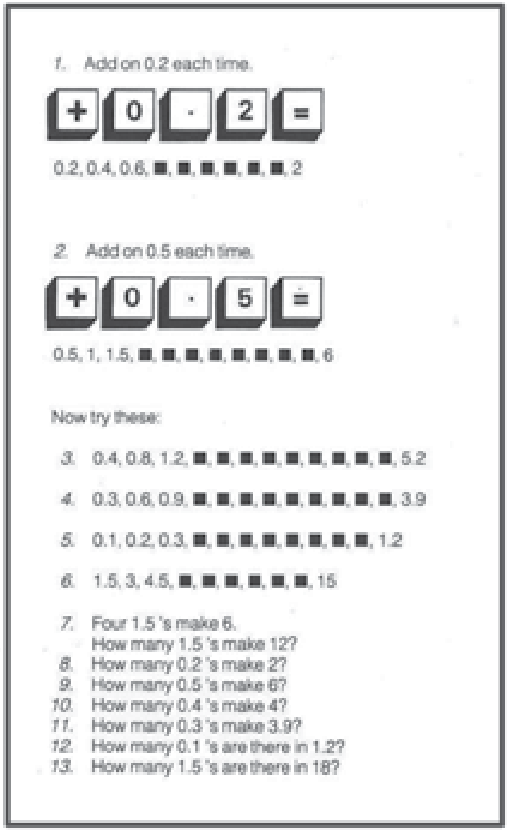
2.
Use arrows to mark the numbers and words on the number line.

Check that your arrows point to the exact number.

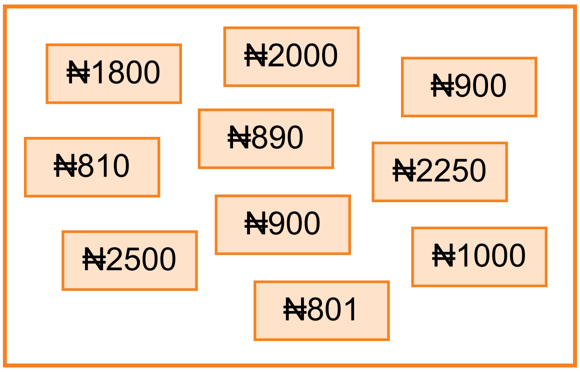


Adding small numbers illustrated as lines, using cm and mm, helps pupils to add decimal fractions which require "carrying" a whole number. Similarly, using a sequence of small decimal steps helps pupils to understand when the next whole number has been reached.

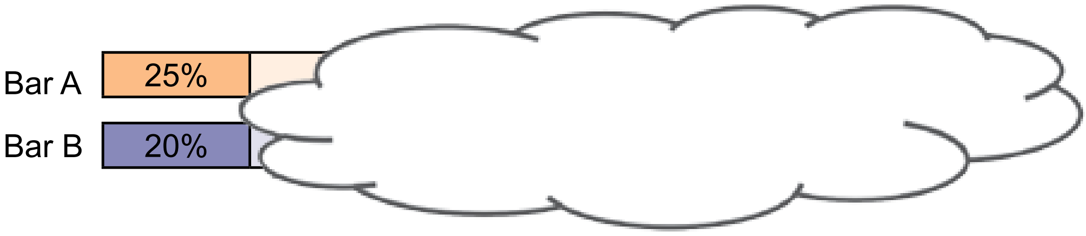




The video extracts from a lesson on percentages showed pupils calculating 10% discounts on prices of several items. A good follow-up lesson would include asking pupils to match 5 pairs of original prices with their prices after a 10% discount.

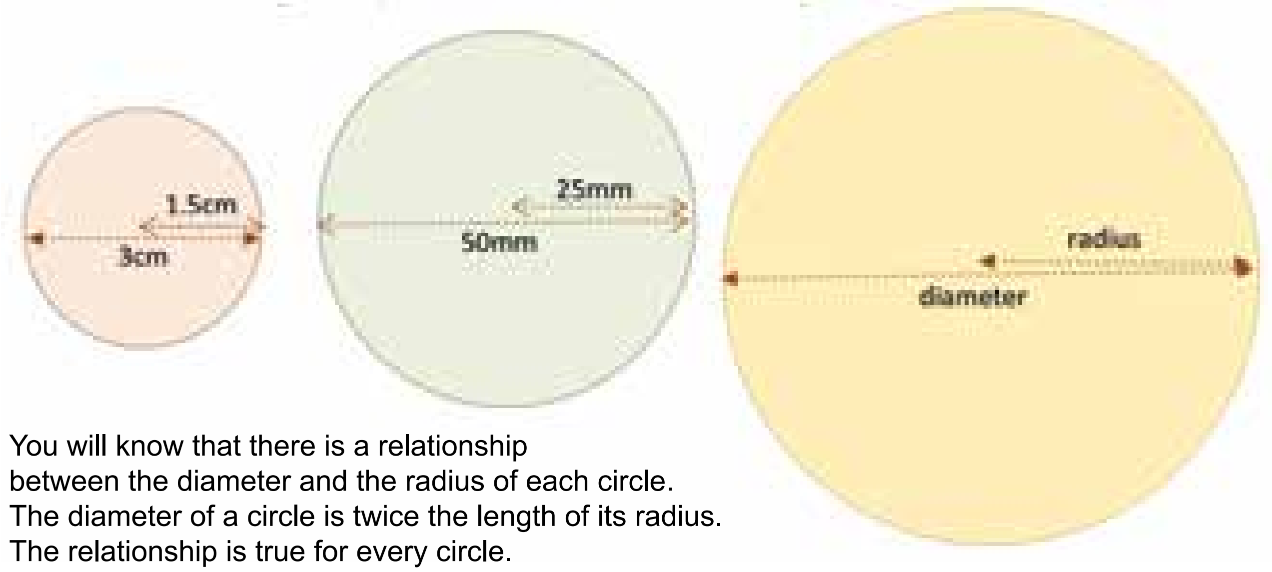


If pupils understand percentages as fractions of 100, they should be able to identify which of these two bars is the longer, even though parts of the bars are not visible:



Linking Number with Algebraic Process

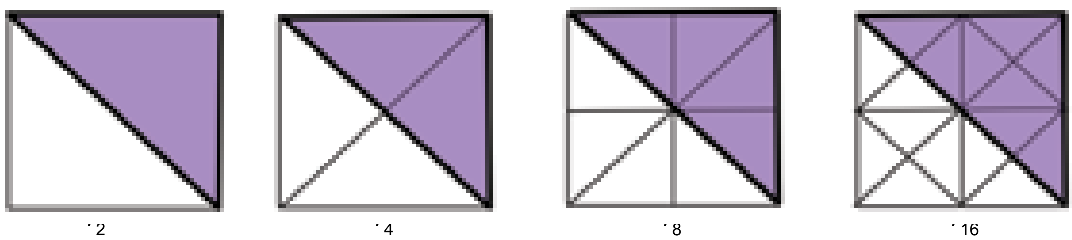
This final section of the Module 7 Summary is a short note about algebraic processes. The essence of an algebraic process is the identification of a generalisation.



You will know that there is a relationship between the diameter and the radius of each circle. The diameter of a circle is twice the length of its radius. The relationship is true for every circle. This generalisation can be expressed in several equivalent ways:

radius = $^1/_2$ diameter	diameter = 2 x radius	radius = diameter ÷2

In the previous sections you have been thinking particularly about fractions. In your lessons about fractions, you will have been encouraging pupils to recognise many different fractions that are equivalent to one-another. For example, you will have been asking pupils to draw many different diagrams which are equivalent to $^1/_2$



You will want pupils to recognise the generalisation that, for all these fractions, the denominator is double the size of the numerator. Recognising that any fraction is equivalent to $^1/_2$ when the denominator is double the numerator, such as $^{25}/_{50}$, $^{75}/_{150}$ or $^{12}/_{24}$, allows pupils to have a greater understanding of the mathematics and to be more proficient with fractions. When the numerator is half of the denominator, then half the number of fraction pieces are shaded. Making such a simple generalisation explicit will help pupils to have a mastery of the mathematics.

What is the relationship between the denominator and the numerator for all the fractions which are equivalent to $^1/_{10}$?

Helping pupils to recognise generalisations is closely linked to the idea of giving pupils ownership of mathematical ideas that we referred to in the introduction of this module.

Experiencing change in the classroom

Were your pupils able to draw and use some of the diagrams that have been suggested as helpful aids to learning about number?

Did you use a number line to help locate improper fractions?

In your teacher's journal, write down your main experiences of trying out some of the suggested activities from this module in your classroom. Questions that might guide your writing about your experiences are:

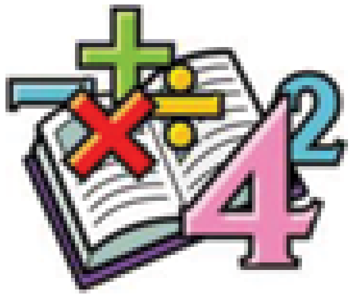
1. Which activities did you try out in teaching these topics in your classroom?
2. Which ones went well? Why?
3. What will you change when you use the same activities or teach these topics next time?

Suggestions for the next cluster meeting

In the empty space below note any topic that you would like to discuss or to share with your Teacher Facilitator or with other fellow teachers in the cluster meeting. These topics can be anything that arises from your experience with Module 7 – for example, an experience of trying out a new activity; a challenge that you want to discuss and find a solution for; something you didn't understand; a question about the mathematics; or any comment that you think might guide other teachers.

Module 8:
Basic Operations

Module 8:
Basic Operations



The basic operation skills – addition, subtraction, multiplication and division – are all important skills that we use in our daily lives. We want to ensure that pupils are able to use these skills to do many calculations mentally. And we will teach them how to use written methods for calculations that would be difficult to do in their heads.

In the previous Teacher Guide (Part B) you have learnt about teaching these skills at a starting level appropriate for P1-3. In this module, we will discuss how to teach these same basic operations but at a higher level appropriate for P4-6. We will ensure that there is a progression from year 1 through to Year 6.

Pupils need to develop both their mental skills and their written skills together so that their mental skills will support their learning of written methods for calculating with bigger numbers, with decimal numbers and with more demanding calculations.

This module focuses on seven skills;

Section 1 covers addition and substraction**:**

Written methods for column addition of whole and decimal numbers	(Years 4-6)
Written methods for column subtraction of whole and decimal numbers	(Years 4-6)

Section 2 focuses on multiplication:

Written methods for short multiplication of HTU and decimals by U	(Years 4-6)
Written methods for long multiplication of TU by TU	(Years 4-5)

Sections 3 looks at Division:

Short division of HTU by U	(Years 4-6)
Multiplying and Dividing by 10 and 100	(Years 5-6)

Sections 4, 5 and 6 each look at one skill

Mental methods for Estimation	(Year 4)
Indices	(Year 6)
The Order of Operations	(Year 6)

Objectives

By the end of this module, teachers will be able to support pupils to:

- add and subtract whole and decimal numbers up to three decimal places;
- multiply 2-digit numbers (including decimals) by a single digit whole number and multiply pairs of 2-digit whole numbers;
- divide 2- or 3-digit numbers (including decimals) by a single digit whole number;
- round numbers to the nearest whole number, ten, hundred or thousand and use these to estimate an answer;
- understand the use of indices for repeated multiplication;
- use brackets and “BODMAS” to ensure efficiency, accuracy and reliability.

Section 1:
Addition of Whole Numbers and Decimal Numbers

The essential understanding for adding and subtracting numbers by a written method depends upon pupils recognising the place value of each digit in any number.

In year 3, pupils will have begun to write calculations using column addition by expanded the numbers (sometimes called “partitioning”).

For example,

$$
\begin{array}{lr}
365 + 274 \longrightarrow & 300 + 60 + 5 \\
& + \; 200 + 70 + 4 \\
\hline
& 500 + 130 + 9 \longrightarrow 639
\end{array}
$$

This builds upon the pupil’s mental skill, recording and making explicit the process by which addition can be done on a number line or in the head.

In Years 5 and 6 pupils will develop and refine this column method without needing to partition the numbers. For some children, this can be difficult because the addition may require crossing the 10, 100 or 1000 boundaries. For example,

$$
\begin{array}{lr}
365 + 74 \longrightarrow & 3\;6\;5 \\
& +\;2\;7\;4 \\
\hline
& 3\;9\;{}_{1}
\end{array}
$$

where pupils next need to recognise that, when adding the 6 and the 7 in this sum, they are actually adding “tens”. The six and seven sum to thirteen – making 13 tens or 130 – that’s 3 tens and 100. They will write 3 in the “tens” column and carry 1 over into the “hundreds” column. This is made more comprehensible to pupils if the column labels are included in the written sum:

$$
\begin{array}{lr}
& \text{H T U} \\
365 + 74 \longrightarrow & 3\;6\;5 \\
& +\;2\;7\;4 \\
\hline
& 3\;9\;{}_{1}
\end{array}
$$

Where the “1” is placed is important. The “1” is marked under the H column so that it is not forgotten or ignored and so that it is not included with “3” in the tens column. Write it under the answer line …

```
  H T U
   3 6 5
 + 2 7 4
 -------
     3 9
 -------
   1
```

… to ensure that the “1” is carried over correctly into the next column. Teachers will always use the same position so that pupils develop a consistent and reliable habit.

The same skill is used in Year 5 for numbers with decimal fraction parts:

```
                          T U   t h
46.45 + 0.78  ------->    4 6 . 4 5
                        +   0 . 7 8
                          ---------
                          4 7 . 2 3
                          ---------
                            1   1
```

In this example, 5 hundredths in the first number and 8 hundredths in the second number add to make 13 hundredths, which is the same as 1 tenth and 3 hundredths. So adding the 5 and the 8 causes a carry-over of “1” into the tenths column. In the next column, 4 tenths and 7 tenths and the extra 1 tenth sum to 12 tenths – which causes a carry-over of 1 to the units column. Because the carry-over goes to the next column on the left, it is important for pupils to learn that they must always start column additions by adding from the right.

Pupils will also learn to extend the column method for adding more than two numbers. For example,

```
                              T U   t h
0.75 + 0.95 + 0.45  ---->       0 . 7 5
                                0 . 9 5
                              + 0 . 4 5
                                -------
                                2 . 1 5
                                -------
                                2   1
```

By Year 6, pupils will know how to write the numbers to be added in the appropriate columns, even when the numbers include a mix of decimal fractions such as

	T	U	.	t	h	th
16.75 + 7.995 + 0.758 + 40 →	1	6	.	4	5	
		7	.	9	9	5
		0	.	7	5	8
	+ 4	0				
	2	5	.	2	0	3
	1	2		2	1	

Here, the important skill is to ensure that the decimal points are lined up so that the numbers are written with each digit in its correct place value column.

Check that you understand why the “2s” and the “1s” in the last two examples have been carried over to the next columns on the left.

Subtraction of Whole Numbers and Decimal Numbers

Subtraction using the column method is not difficult for pupils when the digits of the first number are greater than those of the second number.

For example, in Year 3, pupils were able to subtract 789 – 527 without difficulty by partitioning:

$$789 - 527 \longrightarrow \begin{array}{r} 700 + 80 + 9 \\ -\ 500 + 20 + 7 \\ \hline 200 + 60 + 2 \end{array} \longrightarrow 262$$

Year 4 pupils will easily be able to adapt this to the quicker column addition without partitioning:

$$789 - 527 \longrightarrow \begin{array}{r} \text{H T U} \\ 7\ 8\ 9 \\ -\ 5\ 2\ 7 \\ \hline 2\ 6\ 2 \end{array}$$

If the second number contains larger digits, pupils learnt to re-name the first number so that the subtraction will be possible.

For example:

$$727 - 586 \longrightarrow \begin{array}{r} 700 + 20 + 7 \\ -\,500 + 80 + 6 \\ \hline \end{array} \longrightarrow \begin{array}{r} 600 + 120 + 7 \\ -\,500 + 80 + 6 \\ \hline 100 + 40 + 1 \end{array} \longrightarrow 141$$

Here, because 80 cannot be subtracted from 20, a hundred was taken from the seven hundreds to allow the subtraction to proceed. We now need to help Year 5 and Year 6 pupils to do this same re-naming calculation by the quicker column method, without the partitioning.

The column method will use the same mental logic. Pupils usually find this straightforward unless the re-naming happens several times or if one of the digits in the first number is a zero.

Let us try that last example again, this time without partitioning:

$$727 - 586 \longrightarrow \begin{array}{rccc} & H & T & U \\ & 7 & 2 & 7 \\ - & 5 & 8 & 6 \end{array} \longrightarrow \begin{array}{rccc} & H & T & U \\ & {}^{6}\cancel{7} & {}^{1}2 & 7 \\ - & 5 & 8 & 6 \\ \hline & 1 & 4 & 1 \\ \hline \end{array}$$

You will see that this is, in fact, just a short-hand way of writing the partitioning method.

For this quicker column method, pupils will only need to learn how to write the re-naming so that the change of the digits is recorded. Then they can continue to complete the subtraction.

Ensure that pupils understand why, in the above example, the 7 2 7 has changed to 6 12 7

Subtracting with decimals follows the same procedure…

$$47.84 - 5.27 \longrightarrow \begin{array}{rcccccc} & T & U & & t & h \\ & 4 & 7 & . & {}^{7}\cancel{8} & {}^{1}4 \\ - & & 5 & . & 2 & 7 \\ \hline & 4 & 2 & . & 5 & 7 \end{array}$$

… even when the renaming crosses the decimal point. This is because 1 unit can provide 10 tenths, as in this example:

$$
17.28 - 6.75 \longrightarrow
\begin{array}{rccccc}
 & \text{T} & \text{U} & & \text{t} & \text{h} \\
 & 1 & {}^{6}\cancel{7} & . & {}^{1}2 & 8 \\
- & & 6 & . & 7 & 5 \\
\hline
 & 1 & 0 & . & 5 & 3
\end{array}
$$

Pupils may find this column method difficult at first if the re-naming needs to be done from a zero, as with 2074 – 992

$$
2074 - 992 \longrightarrow
\begin{array}{rcccc}
 & \text{Th} & \text{H} & \text{T} & \text{U} \\
 & 2 & 0 & 7 & 4 \\
- & & 9 & 9 & 2 \\
\hline
 & & & &
\end{array}
\longrightarrow
\begin{array}{rcccc}
 & \text{Th} & \text{H} & \text{T} & \text{U} \\
 & {}^{1}\cancel{2} & {}^{9}0 & {}^{1}\cancel{7} & 4 \\
- & & 9 & 9 & 2 \\
\hline
 & 1 & 0 & 8 & 2
\end{array}
$$

Can you explain how the 17 was obtained?
Where did the 9 in the top row come from?

Here is another column subtraction which Year 5 pupils may find challenging. It is not the decimals that will cause the difficulty; it's the repeated re-naming which can be problematic for some children.

$$
6.725 - 1.995 \qquad
\begin{array}{rccccc}
 & \text{U} & & \text{t} & \text{h} & \text{th} \\
 & 6 & . & 7 & 2 & 5 \\
- & 1 & . & 9 & 9 & 5 \\
\hline
\end{array}
\qquad
\begin{array}{rccccc}
 & \text{U} & & \text{t} & \text{h} & \text{th} \\
 & {}^{5}\cancel{6} & . & {}^{16}\cancel{7} & {}^{1}2 & 5 \\
- & 1 & . & 9 & 9 & 5 \\
\hline
 & 4 & . & 7 & 3 & 0
\end{array}
$$

Can you explain how the 7 became 16?

Think

"When adding or subtracting whole numbers and decimal numbers, it is essential to arrange each number so that its digits are placed under the correct place value heading."

True or False?

"Decimal numbers have two parts: a whole number part and a decimal fraction part."

True or False?

"Addition and Subtraction using the column method must start from the right hand column."

True or False?

Watch

Watch the video clip MM8V1 on your phone.

As you watch the lesson extract, think about how the teacher supports the pupils' learning of addition and subtraction with decimal numbers:

- Does she solve questions by working alone on the chalk board?
- Does she ask questions without offering any explanation?
- Does she explain to pupils each step in adding and subtracting the decimal numbers?
- Does she demonstrate how to write the re-naming of top row numbers to enable the subtraction?

Reflect

- In the video, you saw a group of girls agreeing on the steps to carry out an addition and a subtraction. What had the teacher done to enable this group work?
- A pupil writes

$$36.75 + 77.5 \longrightarrow \begin{array}{r} 3\,6\,.\,7\,5 \\ +\,7\,7\,.\,5 \\ \hline 1\,0\,1\,3\,.\,1\,2\,5 \\ \hline \end{array}$$

How will you help this pupil?

Work with your partner in school

- The following written method for addition adds each of the columns and writes the answers on separate lines:

$$727 + 586 \longrightarrow \begin{array}{rl} \text{H T U} & \\ 7\,2\,4 & \\ +\,2\,6\,3 & \\ \hline 7 & (4 + 3) \\ 8\,0 & (20 + 60) \\ 9\,0\,0 & (700 + 200) \\ \hline 9\,8\,7 & \\ \hline \end{array}$$

Discuss with your partner whether this expanded method of adding 3-digit numbers would be a helpful bridge for some pupils to help their progression from the Year 3 adding with partitioning to the Year 5 adding with the condensed column method.

- A pupil writes

$$20.74 - 9.92 \longrightarrow \begin{array}{cr} & \text{T U . t h} \\ & 21.74 \\ - & 9.92 \\ \hline & 28.22 \\ \hline \end{array}$$

Discuss with your partner how you would help this child.

- Check these lesson plans for more examples to help pupils:
  - P5 Week 2 Addition — Day 1 Adding 2- and 3- digit numbers

    Day 2 Adding with renaming

    Day 4 Adding 3-digit numbers
  - P5 Week 3 Subtraction — Day 1 estimating answers

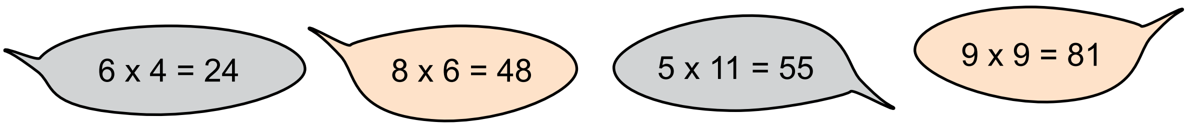
    Day 2 Subtracting 3-digit numbers without re naming

    Day 4 Subtracting with renaming
  - P5 Week 10 Subtraction
  - P6 Week 2 Subtraction
  - P6 Week 11 Addition and subtraction.
- All the examples in this section are simply numbers used to illustrate the column additions and column subtractions. To focus on the method, they are devoid of any real context. Can you and your partner each think of two contexts in which such calculations would be made? For examples,
  - With 627 pupils in the Primary School and 465 pupils in the JSS, how many pupils are there altogether?
  - Efome has 3.85 metres of material. If she uses 1.5 metres for a table cloth, how long is the remaining material?

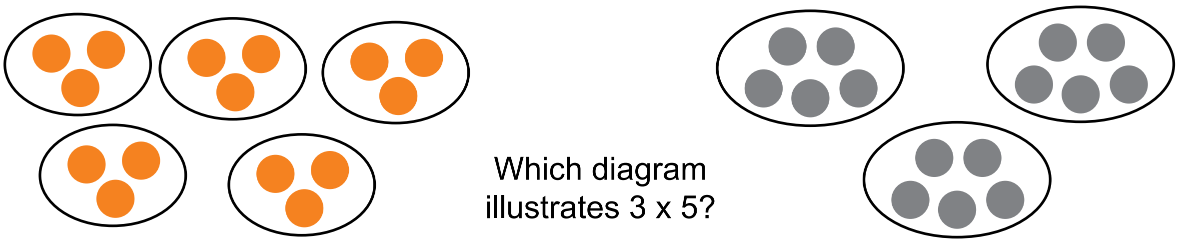
Section 2:
Multiplication

The easiest of the multiplication facts for children to remember is the 10 x table. This will have been learned in Year 2: we will make use of that knowledge here. By Year 4, we will expect that all the children will know, by heart, the multiplication facts for the 2, 3, 4, 5 and 10 times tables. We will also make use of that skill here.

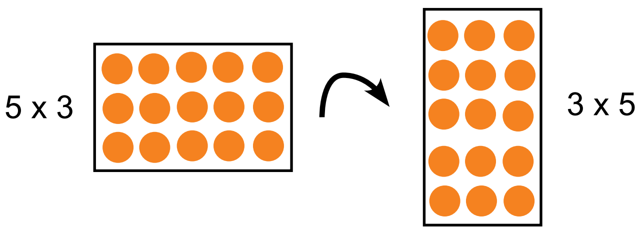
Later multiplication skills are most easily developed and used by pupils who know how to multiply any single digit numbers mentally. Consequently, a lot of time in P4 will be focused on pupils learning and remembering all the multiplication tables so that they will have a quick mental recall of number facts such as 7 x 7 and 6 x 8.



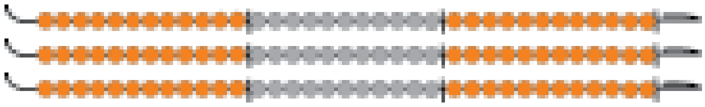
Pupils will know that 3 x 5 is equal to the product of 5 x 3. However, they may have different mental images of what these mean: does 5 x 3 mean five lots of three or three lots of five?



Either interpretation is possible. So to avoid any such difficulty, pupils' learning will benefit from recognising that 3 rows of 5 bottle tops contain the same number of tops as 5 rows of 3.

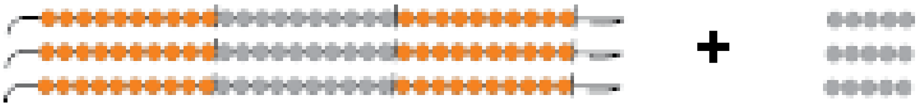


Essentially, the ability to multiply these single digit numbers (or 10 or 11) by another single digit number needs to be a mental skill. When we come to multiplying larger numbers by a single digit number, such as 35 x 3, we will encourage pupils to develop written skills but pupils will still need recall of simple multiplication facts and they will gain from having a mental image of the calculation. You can provide this by using beads or bundles of sticks.



35 x 3

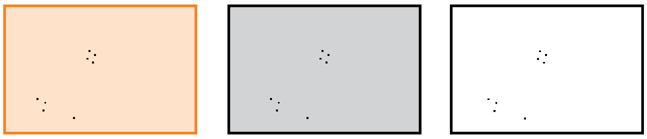
This will help them to develop the understanding that a quick method for writing this multiplication is achieved through partitioning.



30 x 3 5 x 3

Multiply the Tens by 3 and multiply the Units by 3.
Then adding the two parts, 90 and 15, makes the total of 105 easier to calculate.

Using bundles of 10 sticks will also help the children to learn how to write a 2-digit multiplication.

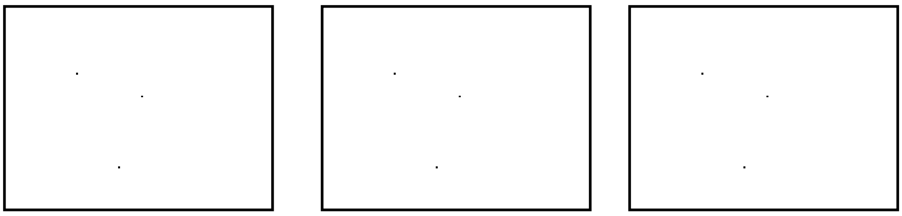


35 x 3 → 30 x 3 and 5 x 3

↓ ↓

90 and 15 → 105

Similarly, you can use bundles to illustrate how to partition 3-digit numbers for multiplication.



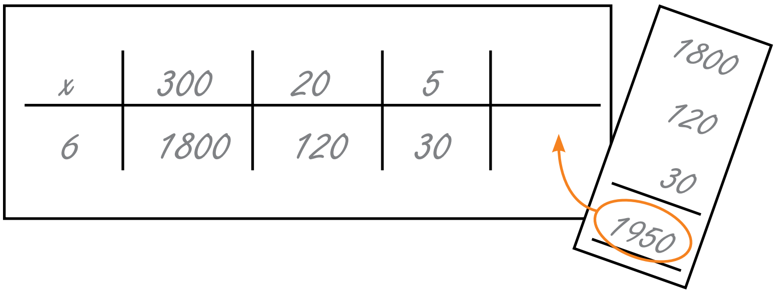
457 x 3 → 400 x 3 and 50 x 3 and 7 x 3

↓ ↓ ↓

1200 and 150 and 21 → 1371

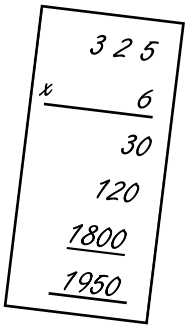
The next step for developing the writing of multiplications is to use the "grid method" (still using partitioning):

325 x 6 →



The separate parts need to be added together to find the total for the calculation. You can see how this grid method leads to the column form of writing this multiplication …

325 x 6 →



… and then to the short multiplication method

$$
\begin{array}{r}
325 \\
\times\ 6 \\
\hline
1950 \\
\hline
\scriptstyle 1\ 3
\end{array}
$$

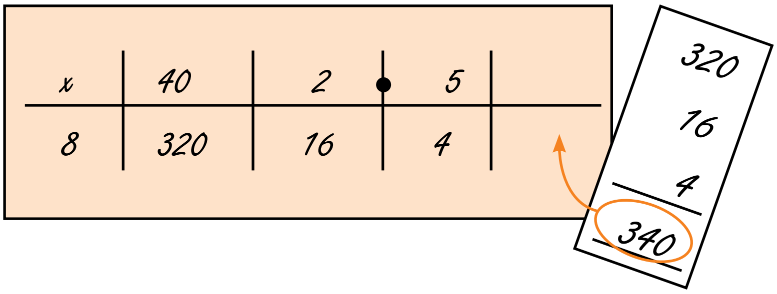
where there is a carry-over of 3 tens into the next column on the left, (and a carry-over of 1 hundred from the tens).

As the classroom teacher, you will need to decide when to introduce each new step in the progression from partitioning to the short multiplication method.

You and your headteacher will be the best people to decide upon how quickly you can progress with the particular children in your class. If you are a good teacher, it is likely that you will differentiate the multiplication activities, having some pupils moving on to the *short multiplication* method while others are still using the *grid method.*

Multiplication of Decimal Numbers

42.5 x 8 →



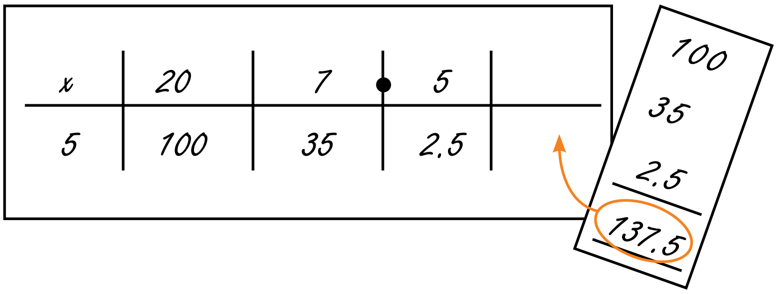
The grid method for multiplying is probably the best way to introduce multiplying when the quantity being multiplied is a decimal number.

The grid method allows pupils to get used to dealing with the decimal part of the number when it crosses the decimal point, the boundary between the whole number and the fraction part.

But, as you will see with the following example, 27.5 x 5, when moving from the grid method to the short multiplication method, the procedure is just the same as when there is no decimal fraction part.

The grid method

27.5 x 5 →



The short multiplication method

H T U t
2 7. 5
x 5

1 3 7. 5
32

Note that there is a carry-over of 20 tenths as 2 into the unit column on the left, because 25 tenths = 2.5

The grid method " illustrated example with "The short multiplication method" example to allow for missing text (para beginning "We only need to remind you.

Multiplication by a 2-digit number

Pupils in year 5 will extend their use of the grid method to help them to multiply by 2-digit numbers. The multiplier, 15 in the following example, is partitioned into Tens and Units in a way similar to how the quantity 43 is partitioned.

43 x 15 →

x	40	3	
10	400	30	
5	200	15	

430
+ 215
645

The 43 is multiplied in the first row by the Ten and then, in the second row by the Unit. You can see that the fourth column in the grid will contain the results of multiplying 43 by 10 and of multiplying 43 by 15. These two partial results need to be added.

We will use the next example to show how the grid method leads to the long multiplication method.

56 x 23 →

x	50	6	
20	1000	120	1120
3	150	18	168
			1288

There are four separate multiplications to be condensed into the one long calculation. First of all, as with the grid, the 50 and the 6 are multiplied by 20. Then the 56 is multiplied by 3. This is also easier if it is done in two parts.

```
    5 6
x   2 3
1 1 2 0
  1 6 8
1 2 8 8
```

56 x 20

6 x 20 = 120
and
50 x 20 = 1000

1120

THAT'S 120 + 1000 MAKING 1120 ALTOGETHER.

56 x 3

6 x 3 = 18
and
50 x 3 = 150

168

THAT'S 18 + 150 MAKING 168 ALTOGETHER.

As pupils become familiar with this long multiplication method, they will be able to work the four separate multiplications mentally. In Year 6, pupils learn how to multiply the tens and units columns digits separately to condense this process even further. Their mental skills will be used to avoid the extra calculations and to write the results directly within the calculation, sometimes carrying digits into a column on the left.

The TDP Lesson Plans for Year 6 Week 4 Multiplication show this final stage in the progression of learning about how to multiply efficiently using the written method of long multiplication. Note that its efficiency depends upon pupils having quick recall of their multiplication table facts - a key skill which requires frequent re-inforcement via your Daily Practice sessions.

Think

- Do you visualise 2 x 7 as “two groups of 7” or as “7 pairs”? What difference might this make to your pupils’ understanding of multiplication?
- Have you recognised a progression from partitioning to the long multiplication method. What role does the grid method play in this progression?
- Do you teach “x 10” as a multiplication by a 2-digit number or do you expect children to be able to multiply by 10 as a single mental recall?

Watch

Watch the video clip MM8V2 on your phone.

As you watch the lesson extract, name two things that you liked about the lesson extract and name one thing you would like to improve.

Reflect

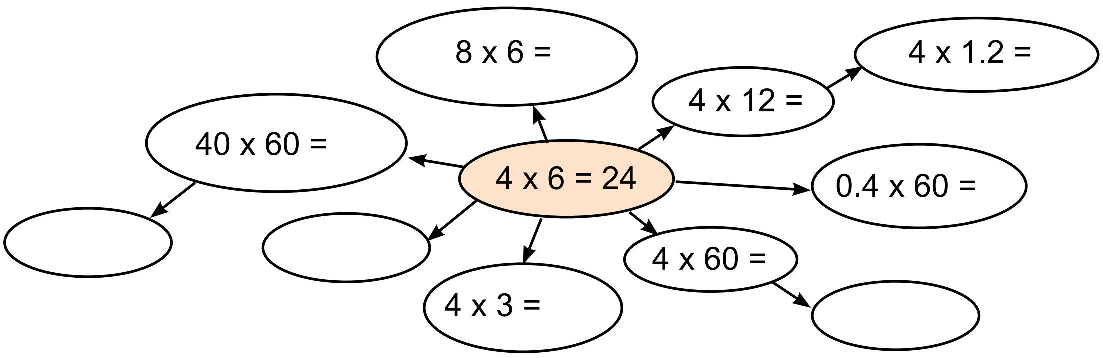
- In the video, you saw two pupils explaining how to multiply pairs of 2-digit numbers. Hamisu shows the steps for multiplying 34 x 21 and Nafisa shows the steps for multiplying 25 x 33.

  What short cuts would you advise Hamisu and Nafisa to take?
- In the long multiplication method for 56 x 23 (shown on the previous page) would it make any difference if you multiplied 56 x 3 first and then multiplied 56 x 20 ?

Work with your partner in school

- Start with a simple multiplication such as 4 x 6 = 24

  What other multiplications can this lead to?



Plan a lesson in which you ask each group of pupils to choose a multiplication that they know as a starting point for other multiplications. “If you know … what else can you know?”

- Discuss with your partner whether you would introduce the grid method for multiplying a decimal number before or after introducing the short multiplication method for whole numbers.

- Check these lesson plans for more examples to help pupils:

• P4 Week 1	Multiplication	Day 1 Multiplying by 10 and 100 Days 2,3 Multiplication using the grid method Days 4,5 Multiplication word problems
• P4 Week 16	Multiplication	Day 1 The grid method Days 2,3 Multiplying decimal numbers
• P5 Week 4	Multiplication	Day 1 Grid Method for HTU x U Day 2 Multiplying 2-digit numbers Day 3 Multiplication using the grid method
• P6 Week 4	Multiplication	
• P6 Week 12	Multiplying with decimal numbers.	

Section 3:
Division

Division in year 3 is done with the aid of objects such as seeds, bottle-tops, sticks and stones. At this stage, pupils are only writing the result of physically sharing a quantity equally, sometimes with remainders. This recording of a division is formalising an activity with which they will be familiar at home where they share things with their brothers and sisters.

In Years 4 and 5, pupils learn how to calculate a division using more formal methods. They will use diagrams and writing skills to divide numbers with up to four digits by a single digit number. As you will see, the ability to multiply by 10 and by 100 will be a useful mental tool to help pupils to write their division calculations. Note that multiplying and dividing by 10s and by 100s is usually treated separately because the knowledge associated with multiplying and dividing by 10 and by 100 leads to important lessons about place value and decimal numbers.

Year 6 pupils will learn how to divide by 2-digit numbers, extending their use of the formal written method called long division. The short division skills to be learnt in Year 4 are a key step in beginning this progression.

Let us start this Year 4 work with three problems that might be familiar to the pupils.

Safiya has 24 sweets to share equally between herself and 2 sisters.

How many sweets will each of the 3 girls have?

There are 12 children without seats in the classroom.

How many more benches are needed?

Each bench can seat 4 children.

Abiola's father has many hens and he sells the eggs they produce in egg boxes.

The egg boxes hold 6 eggs each.

Will 50 eggs be enough to fill 8 egg boxes?

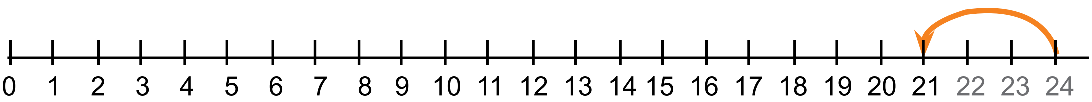
Answer the three questions and think about how you solved them.

At first glance these look like any collection of word problems about division. However, each different scene draws attention to a different way of thinking about the idea of division:

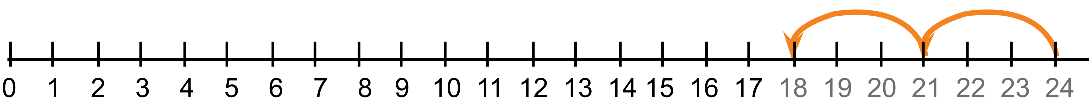
- sharing, grouping *(In Q1 you divided the 24 into 3 equal groups of 8*.)
- successive subtraction *(Q2: Twelve take 4, three times;* or *how many fours in 12?)*
- the inverse of multiplication (*Q3: 8 x 6 = 48, so 50 eggs will be more than enough*.)

Question 1 can be modelled for your pupils by sharing 24 bottle-tops or seeds. When pupils try this, ask them to find an efficient strategy for the sharing. Some pupils will want to do this by allocating seeds, one-at-a-time, to 3 people. Others may allocate two or more at a time to 3 people until all the seeds have been used. There is no easy way of writing down the sharing process that pupils are engaged in, other than to record the result 24 ÷ 3 = 8

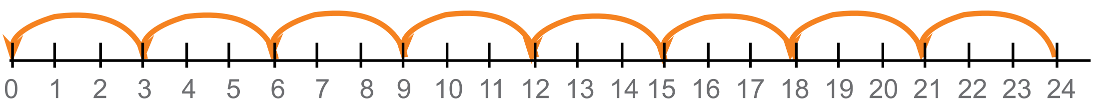
However, a number line offers a visual way of recording the sharing of the 24 sweets. Giving each girl one sweet each uses up 3 of them. There are 21 sweets remaining.



Giving out another round of 3 sweets, there will be 18 left.

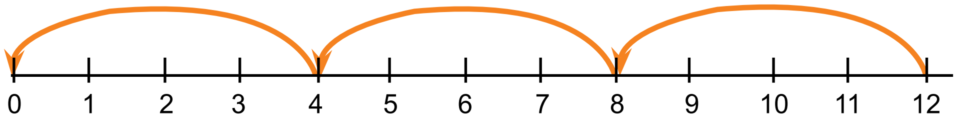


The sharing by subtraction continues until there are no sweets left.



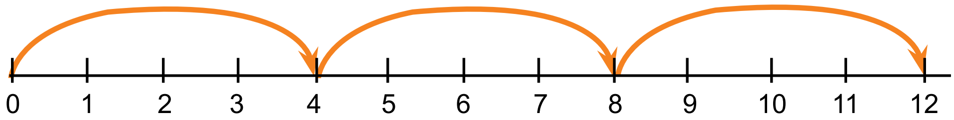
Eight subtraction steps are needed. So the number line shows that the division of 24 by 3 is 8.

A number line can also illustrate that 3 benches are needed in Question 2.



For 12 pupils who have no seat, 4 of them can sit on 1 new bench. That leaves 8 pupils with no seat. Another 4 can sit on a second new bench, leaving 4 who will need a third new bench.

In finding out how many 4s reach 12 (or how many 3s make 24) the calculation is, in fact, showing the inverse process of multiplication.



4 pupils need 1 bench. 8 pupils need 2 benches. 12 pupils need 3 benches.

Using the number line in this way, you can see that when you divide 12 by 4, you are asking how many 4s will make 12. For 24 ÷ 3, you were asking how many 3s make 24. So the ability to do a division is based upon recognising a multiplication.

When calculating 30 ÷ 5, your answer will tell you how many 5s make 30. Therefore, to divide 30 ÷ 5 without using a number line, you need to know that 5 x 6 = 30

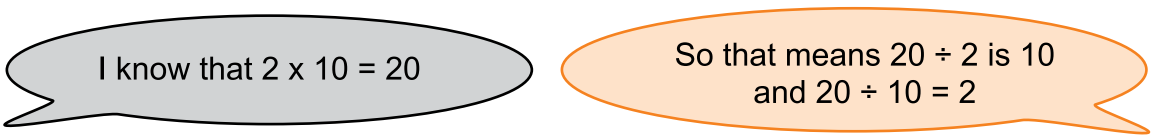
This means that, after illustrating some divisions as repeated subtractions on a number line, the next activity for Year 4 pupils will be to recognise that being able to calculate a division depends upon knowing a multiplication.

"If you know that 3 x 5 = 15, then you know that 15 ÷ 3 = 5 and 15 ÷ 5 =3"

The diagrams showing that 3 x 5 = 15 in Section 2 Multiplication also show 15 ÷ 3 and 15 ÷ 5

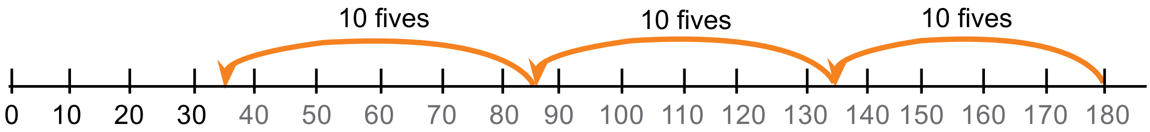
"Tell your partner a multiplication fact that you know.

Your partner will tell you two related divisions that she knows."

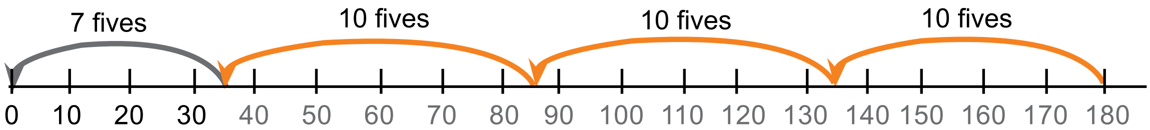


Knowing such divisions depends upon pupils having mental recall of their multiplication tables. Consequently, dividing quantities up to 100 will often be manageable as a mental activity.

Dividing larger quantities will be more challenging and involves more steps. To divide 185 ÷ 5, for example, pupils can use their knowledge that 5 x 10 = 50 This will help them to recognise that 5 x 30 = 150. But the 150 is still not enough to reach 185 – another 35 is needed.



Pupils will see that the remaining 35 can be made with 7 fives



In Year 5 pupils will begin to learn how to write this repeated subtraction without the number line.

This is the "long division" format.

So 185 ÷ 5 = 37

```
    37
5 | 185
    150 ←— 5 x 30
     35
     35 ←— 5 x 7
      0
```

Here are two more examples of the long division method, written without a number line:

78 ÷ 6
where pupils will use 6 x 10 = 60 as a first step:

```
6 | 78
    60 ←— 6 x 10
    18
```

The remaining 18 can be divided by 6, 3 times:

```
    13
6 | 78
    60 ←— 6 x 10
    18
    18 ←— 6 x 3
     0
```

So 78 ÷ 6 = **13**

92 ÷ 4
where pupils can use 4 x 10 as a first step:

```
4 | 92
    40 ←— 4 x 10
    52
```

The remaining 52 will allow another 40 (4 x 10) to be subtracted…

```
4 | 92
    40 ←— 4 x 10
    52
    40 ←— 4 x 10
    12
```

…and that leaves a remaining 12 which allows another 3 fours to be subtracted

```
    23
4 | 92
    40 ←— 4 x 10
    52
    40 ←— 4 x 10
    12
    12 ←— 4 x 3
     0
```

So 92 ÷ 4 = **23**

Can you shorten this division by taking a bigger first step ?
Try using 4 x 20 = 80

Long division for decimal numbers follows the same "chunking" procedure.
Compare these two long division calculations:

572 ÷ 4
where pupils can use 4 x 100 = 400 as a first step:

$$\begin{array}{r|l} 4 & \overline{572} \\ & \underline{400} \leftarrow 4 \times 100 \\ & 172 \end{array}$$

The remaining 172 will allow another 160 (4 x 40) to be subtracted…

$$\begin{array}{r|l} 4 & \overline{572} \\ & \underline{400} \leftarrow 4 \times 100 \\ & 172 \\ & \underline{160} \leftarrow 4 \times 40 \\ & 12 \end{array}$$

…and that leaves a remaining 12 which allows another 3 fours to be subtracted

$$\begin{array}{r|l} & 143 \\ 4 & \overline{572} \\ & \underline{400} \leftarrow 4 \times 100 \\ & 172 \\ & \underline{160} \leftarrow 4 \times 40 \\ & 12 \\ & \underline{12} \leftarrow 4 \times 3 \\ & 0 \end{array}$$

So 572 ÷ 4 = **143**

57.2 ÷ 4
where pupils can use 4 x 10 = 40 as a first step:

$$\begin{array}{r|l} 4 & \overline{57.2} \\ & \underline{40} \leftarrow 4 \times 10 \\ & 17.2 \end{array}$$

The remaining 17.2 will allow another 16 (4 x 4) to be subtracted…

$$\begin{array}{r|l} 4 & \overline{57.2} \\ & \underline{40} \leftarrow 4 \times 10 \\ & 17.2 \\ & \underline{16} \leftarrow 4 \times 4 \\ & 1.2 \end{array}$$

…and that leaves a remaining 1.2 to be divided by 4. We will use 4 x 0.3 = 1.2

$$\begin{array}{r|l} & 14.3 \\ 4 & \overline{57.2} \\ & \underline{40} \leftarrow 4 \times 10 \\ & 17.2 \\ & \underline{16} \leftarrow 4 \times 4 \\ & 1.2 \\ & \underline{1.2} \leftarrow 4 \times 0.3 \\ & 0 \end{array}$$

So 57.2 ÷ 4 = **14.3**

Can you draw a number line to illustrate the three steps of this long division method?

Pupils will need plenty of practice at using repeated subtractions for division before writing these long division calculations themselves. They need to discover what size steps to subtract and they need to discover that the process is quicker when they subtract bigger steps.

Pupils will find jumping down in 10s the easiest way of taking big steps. You can encourage pupils to take bigger steps by jumping in multiples of 10 so that they become familiar with jumps of 20 or 40 or 50.

How many 40s make 200?

How many 10s are needed to make 100?
How many 10s are needed to make 200?

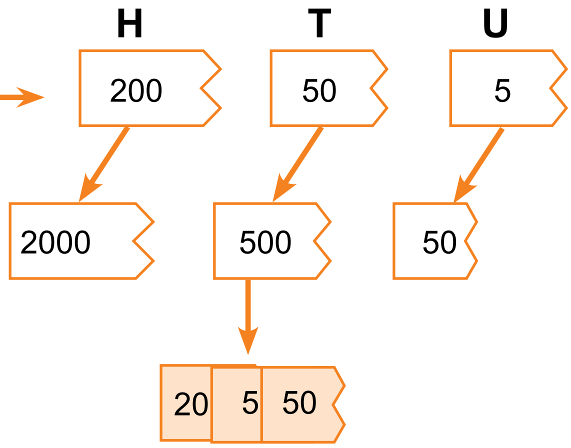
Divide 200 Naira into 50s.
How many 50 Naira notes will be the same as 200 Naira?

Division by 10 and by 100

As you know, dividing is the inverse processes of multiplying. So, first of all, make sure that you understand what happens when you multiply by 10 and 100.

Look at 255 x 10

Split the number 255 into its parts…

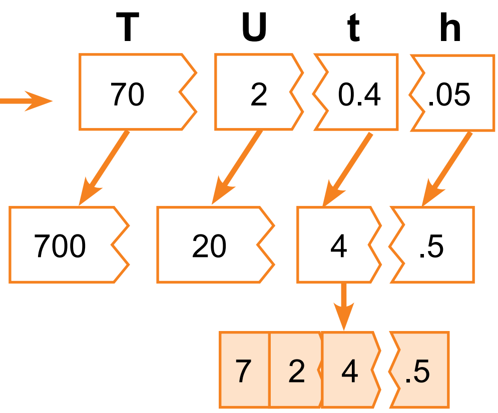


…and multiply by 10

You can see that, when 255 is multiplied by 10, each part gets ten times bigger to make 2550. The HTU column headings for the digits become ThHT

255 x 10 = 2550

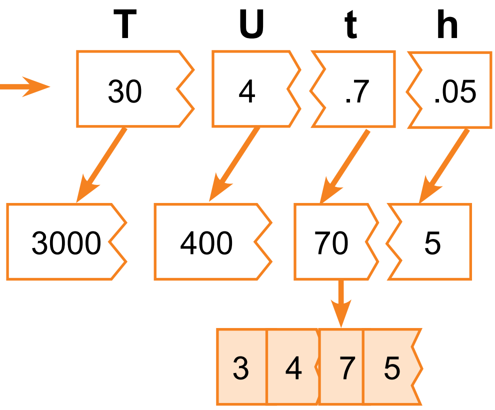
For a number with a decimal point, the same thing happens.
Look at 72·45 x 10



… multiplying by 10 makes each part ten times bigger and the product is 724·5

When you compare the numbers 72.45 and 724.5 it looks like multiplying by ten has caused the decimal point to move one place to the right. The truth is that, in fact, the decimal point has remained in the same place and each of the digits has moved one place to the left. When 72.45 has been multiplied by 10, the 70 has become 700; the 2 has become 20; the 4/10 has become 4; and the 5/100 has become 5/10. Each part has become ten times bigger and so each digit has moved to the next higher place on its left.

Here is another example – this time for multiplying by 100



34·75 x 100
Multiplying by 100 makes each part 100 times bigger.
34·75 x 100 = 3475
Each of the digits has moved two places to the left. So it looks like the decimal point has moved two places to the right.

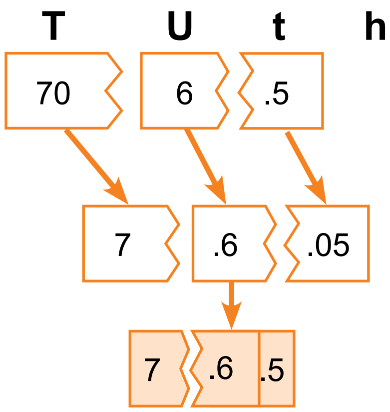
Multiplying by 1000:

63.5 x 1000 = 63500

If the decimal point is included in this answer, we would write 63500**.** But it is not necessary because there are no decimal parts after the decimal point.

Why does it look like the decimal point moves three places to the right when multiplying by 1000?

The opposite moves happen when you **divide** a decimal number by 10
Divide 76.5 ÷ 10



Dividing by 10 makes each part ten times smaller…

The T U t column headings for the digits become U t h

76·5 ÷ 10 = 7·65

Each digit has moved to the next smaller place on its right.

So it looks like the decimal point has moved one place to the left. When each digit has moved one place to the right, the number is ten times smaller or a tenth of its original size.

So that pupils will understand how a number changes when they divide by 10, use columns to emphasise that it is the digits which change position. It looks like the decimal point moves (and, in practice, most people will change the position of the point when they divide decimal numbers) but remember that it is the digits which move! This will be particularly important for Year 6 but it is also important that pupils in Year 4 and Year 5 do not receive a wrong message.

$$76{\cdot}5 \div 100 = 0{\cdot}765$$

Why do you think that it looks like the decimal point moves two places to the left when you divide by 100?

Here is a useful Year 6 activity. Our next Section on Estimating will make this task easier.

Can you match each of these calculations with their correct answer:

25 x 10	2.5
25 ÷ 10	250 000
2500 x 100	250
2500 ÷ 100	25
7.5 x 10	0.75
7.5 ÷ 10	7.5
750 x 10	75
750 ÷ 100	7500
3.333 x 10	0.3333
3.333 ÷ 10	0.03333
3.333 x 100	33.33
3.333 ÷ 100	3333

Think

When you have taught pupils to divide 27 ÷ 3, have you used subtraction or multiplication to help pupils to calculate the answer?

Watch

Watch the video clip MM8V3 on your phone.
As you watch the lesson extract, notice how the teacher illustrates the long division chunking on the number line.

Reflect

In the video, how did the teacher encourage children to choose the size of chunking to subtract?

Did she allow different children to choose different size steps or did she expect everyone to use the same size steps?

Work with your partner in school

Illustrate the division 105 ÷ 5 on a number line. What steps do you use?
Discuss whether this division should be a mental activity.
Match each of these multiplications and divisions with the correct answer.

1.	2·05 x 10		A	0.205
2.	2·05 x 100		B	2.05
3.	2·05 ÷ 10		C	20.5
4.	205 ÷ 10		D	205
5.	205 ÷ 100			

Check the lesson plans for more examples to help pupils:

• P4 week 5	Division	Day 1 Number Line
		Days 2,3,4 Repeated Subtraction
		Day 5 Division by 10
• P4 week 17	Division	Day 1 Multiplication for Division
		Day 2 Repeated subtraction “chunking”
		Day 3 Remainders
		Day 4 Chunking
• P5 week 5	Division	Day 1 Repeated subtraction (TU ÷ U)
		Day 2 Repeated subtraction (HTU ÷ U)
		Day 3 Dividing by 10, by 100
• P5 Week 16	Division	Dividing by 10 and 100
• P6 Week 5	Division	
• P6 Week 13	Division with decimal numbers	

Section 4:
Estimation

For many situations, an exact answer is not possible or is not appropriate.

If Ibrahim arrives home after 34 minutes, having taken a car to travel the 13½ km to the market, after a flight that took 5 hours 50 minutes, he will have made good estimates of the distance and the times. He gave answers that were sensibly close to the actual figures.

The skill of estimating a sensible answer is just as important in mathematics as it is in your home life - even when an exact answer is needed. Being able to estimate an answer enables you to know how to solve a problem, as well as knowing that your answer is a reasonable one.

Estimating Measures

- How wide is your classroom? 3 metres? 5 metres? 10 metres?
- How tall is the tree? 5 metres ? 10 metres ?
- How heavy is the sack of ground-nuts? 20kg ? 50kg ?
- How many grams is a teaspoon of salt? 1g ? 5g ? 10g ?

To be able to estimate measurements, you need to have a rough idea of the size of all our basic units for measuring length, mass, volume and time. The key units are *metre* for length (m), *gram* for mass (g), *litre* for volume (l) and *hour* for time.

Your pace is about 1 metre long.
This page is about 20 centimetres (cm) wide and 30 centimetres in length.
Your hand span is about 20cm. Your finger is about 2cm wide. Here is 1cm:

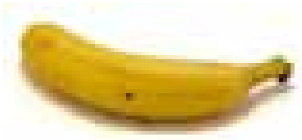
This teaspoon holds 5g of sugar.

- A 1 litre bottle of drinking water weighs 1 kilogram.
- A can of Coca Cola contains about 300 millilitres.
- A bucket of water can hold between 10 or 20 litres; it depends whether your bucket is small or large.

To be able to estimate measures, pupils will need experience of using the measurements: pacing across the classroom, holding a kilogram weight, walking a kilometre, counting 60 seconds to mark a minute, and so on. None of these are easy concepts to teach but when pupils calculate with any measures, they need to develop a sense of size. They should be able to estimate the size of any object, choosing the best unit to describe that size.

The length of a banana is about …

- ☐ 2 cm
- ☐ 20 cm
- ☐ 200 cm
- ☐ 2000 cm



Estimating Calculations

Pupils also need to develop a sense of size for the calculations that they do in Mathematics.

In the same way that Ibrahim made a sensible estimate of the time that his journey would take, children will learn how to estimate the size of the answer to a problem or calculation. Anticipating the size of an answer helps them to know what calculation is needed and whether the answer is appropriate. They should know that adding makes quantities larger and that subtracting makes things smaller. Multiplying increases the size or number of items; dividing reduces the number of items in a group (unless you are multiplying and dividing by fractions).

Musa wants to know if his truck, which can carry a maximum of 1 tonne (1000 kg), can transport 210 blocks which each weigh 3.8 kg. To calculate 210 x 3.8 he recognises that the operation is a multiplication, so he knows that the answer will be bigger than 210.

The key skill for estimating is to be able to round each number to a near convenient amount. Rounding will enable Musa to know that the answer for 210 x 3.8 will be close to 800kg because the multiplication 210 x 3.8 is close to 200 x 4. So Musa’s estimation helps him to know that he can safely carry all the blocks in one trip.

Here are seven calculations for you to consider some principles about estimating:

# 1	Before you start the calculation decide whether the answer will be bigger or smaller.	**20.8 ÷ 4** Dividing is going to make the answer smaller than 20.8		
# 2	Round the numbers to the nearest convenient size H, T or U for the calculation.	**20.6 x 3.9** Choose 20 x 4 The answer will be close to 80		
# 3	Choose the rounding so that a mental calculation will be possible.	**20.6 ÷ 3.2** Choose 21 ÷ 3 rather than 20 ÷ 3 because you know that 21 is divisible by 3 but 20 isn't. This answer will be close to 7		
# 4	In estimation, the first digit is important but look at the second digit to decide if you will round up or round down.	**322.2 + 2709** Add 300 and 2700 to get an estimate of 3000		
# 5	If you have many similar numbers to add, choose a convenient average to represent each one.	**245 + 280 + 215 + 272 + 230** There are 5 numbers, all close to 250. So 5 x 250 will give a good estimate.		
# 6	Group together numbers that will make the calculation easy to do.	**76 + 49 + 22 + 53** 76 + 22 will be close to 100 49 + 53 will also be close to 100 So the actual answer will be close to 200		
# 7	For decimals, percentages and fractions, choose an approximation to make the estimate easy to do mentally.	1.4 x 20 $1\frac{1}{2}$ would be a good rounding to choose because $1\frac{1}{2}$ x 20 can be done in your head to estimate an answer close to 30	21% of ₦150 20% is $\frac{1}{5}$ so a good estimate would be ₦30	$\frac{4}{5} + \frac{7}{8}$ Each fraction is close to one so the answer must be close to 2

Think

When you have asked pupils to carry out a calculation, have you first asked them to predict whether their answer will be larger or smaller? How have you asked them to anticipate the size of the expected answer?

We will focus on Measurements in Module 10. In the meantime, have you asked pupils to use any of their own body measurements to estimate a length, such as arm-length, pace, foot or hand-span?

Watch

Watch the video clip MM8V4 on your phone.

You will see the first part of a lesson in which the teacher asks pupils to estimate. She asks the pupils to write out in detail the thinking that it is necessary for them to do in their heads when estimating. She wants to make the mental process explicit.

Her purpose is also to allow the children to have confidence that their ability to estimate will give answers which are reliably close to the actual answer.

Reflect

After the pupils have gained confidence in their ability to make good estimates for a calculation, do you think that the second part of the lesson will have asked pupils

- to do another exercise in the same way ?
- to do group work activities in the same way ?
- to do an exercise of calculations where pupils only write estimated answers?
- to do group work where pupils only write estimated answers for the questions given and then swap answers with other groups to decide whether their estimates are sensible and reasonable ?

Work with your partner in school

If, in the example of Musa and his truck, he had multiplied 210 x 3.8 exactly, the answer would have been 798. Well, he already expected that it would be close to 800 and so this exact answer will confirm that his estimated answer was sensible and appropriate. But would there have been any purpose in Musa estimating an answer if he intended to calculate it exactly?

Discuss with your partner why and when pupils would need to make an estimate.

Although you will study the teaching of Measurement in Module 10, you may like to look now at the poster “Units of Measurement”. It will provide some support for estimating quantities using the standard units of *centimetre, metre* (and *kilometre*), grams (and *kilograms*) and *litres*. Discuss with your partner how you could use the poster to help pupils to make sensible estimations for the sizes of a variety of things, like the height of your classroom, its area or its volume; the mass of a book; …

Finally, regarding remarks #3 and #4 on the previous page, note that pupils will learn the skills of rounding up and rounding down to different degrees of accuracy in JSS.

Section 5:
Indices

For many situations, an exact answer is not possible or is not appropriate.

Indices are a mathematician's shorthand. Instead of writing 10 x 10 x 10 x 10 mathematicians write 10^4. The index ^{4} tells you how many tens have been multiplied together.

- Can you explain why 10^3 is one thousand ?
- Why is 1 million the same as 10^6 ?

The index is usually called the power of the base number.

So 1 trillion (10^9) is called " ten to the power of nine". Ten is the base number.

Note that indices are only used for repeated multiplication.

- Why does 5^2 (five to the power of two) have the value of twenty-five and not ten?
- Explain why 4^2 is not 8

Year 6 pupils will have met repeated multiplication when factorising numbers such as 48.

48 = 2 x 2 x 2 x 2 x 3

This information can be abbreviated to $4^8 = 2^4$ x 3

Using the index notation makes the writing of the repeated multiplication much briefer.

- Using the power notation, how would you write 144 = 3 x 3 x 2 x 2 x 2 x 2 ?
- What number is equal to 2^3 x 5^2 ?

Did you agree that 2^3 x 5^2 is 200 ?

In such work, pupils will often say that 2^3 is 6 because they will think, correctly, that 2 is to be multiplied by itself 3 times but the notion of "2" and *"three times"* may lead them to say, incorrectly, 6 instead of 8.

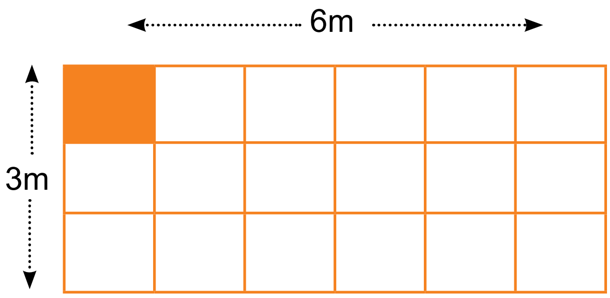
You will need to remind pupils that the index 3 tells them how many times the 2s must be multiplied together: "2, times 2, times 2 again".

" 2 x 2 equals 4, times 2 again, equals 8"

Check that you agree that 2^5 is 32.

Indices are also used with the units for measuring length.

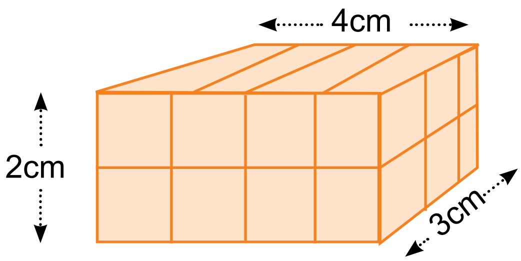
When you calculate the area of a rectangle 3m x 6m your answer will be $18m^2$ because the unit of measurement for area is a 1m x 1m square. You will have multiplied metre x metre and so the unit for area is m2 "a square metre"

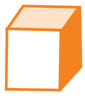


Area = $18m^2$

$1m^2$ - a metre square or "square metre"

A football field 100m x 50m has an area of 5000 m^2. To calculate this you have multiplied 100m x 50m. The *metre* has been multiplied twice (m^2) to describe the 2-dimensional unit for area.





$1cm^3$ - a centimetre cube or "cubic centimetre"

When measuring volumes, the unit of length will be multiplied three times to describe the size in the 3-dimensional units.

This small cuboid box 4cm x 3cm x 2cm has a volume of 24cm3. To calculate the volume the *cm* of width are multiplied by the *cm* of height and by the *cm* of length, so the unit for measuring small volumes is the *cm3.*

Because of the use of squares and cubes in measuring areas and volumes, the words "*square*" and "*cube*" are commonly used instead of the phrases "to the power of two" and "to the power of three". So the area of the rectangle is 18 m2 - "*eighteen square metres*" and the volume of the box is 24cm3 - "*twenty-four cubic centimetres*".

With numbers, we often say 5^2 is *"five squared"* instead of *"five to the power of two"* and 100 is "*ten to the power of 2"* or *"ten squared".* Similarly, 27 is "*three cubed*" because 3^3 is 27.

Pupils will enjoy discovering mathematics if they are given the chance of discovering the laws of indices themselves.

- Ask pupils to describe what happens when numbers with powers of the same base are multiplied together.

They will tell you that $2^5 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$
and $10^2 \times 10 = 10 \times 10 \times 10 = 10^3$
and so they know that to multiply numbers written with powers of the same base, you just need to add the indices to simplify the answer.

- Tell them to investigate a rule for dividing numbers written with powers of the same base:

$$2^8 \div 2^5 = \frac{{}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{2} \times {}^{1}\cancel{2} \times 2 \times 2 \times 2}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}} = 2 \times 2 \times 2 = 2^3$$

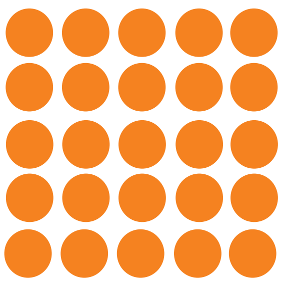
- Able pupils may like to investigate a rule for squaring or cubing a number with an index (but this is not a requirement of the Primary School curriculum)

$$(10^2)^3 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

Pupils will meet the square numbers, 2^2, 3^2, 4^2, 5^2, and so on .. very frequently in the later mathematics of geometry and algebra. So it is important for Year 6 pupils to be able to calculate and recognise the square powers of all the numbers from 1 to 15.

They will recognise 5^2 is 25 both by the illustration 5 x 5 and by calculating 5 x 5.

Equally important is the ability to recognise the square root of the square number.



The square root of 25 is 5. The square root is the number which, when multiplied by itself, makes the square number.

The square root of 100 is 10 because $10^2 = 100$.

Think

The indices in the multiplication $10^4 \times 10^2$ are 4 and 2

What are the indices in the multiplication 10×10^3?

Watch

Watch the video clip MM8V5 on your phone.

As you watch the short lesson extract, think about the examples which the teacher uses to introduce the indices. What contexts did he use to show indices being used?

Reflect

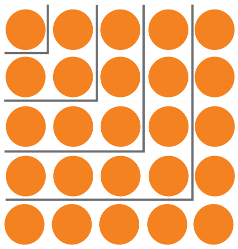
In the video, the teacher uses the pupils' experience of factorising numbers such as 100 to introduce indices. Why do you think this was a good strategy?

Work with your partner in school

- Investigate the number pattern created by the square numbers.

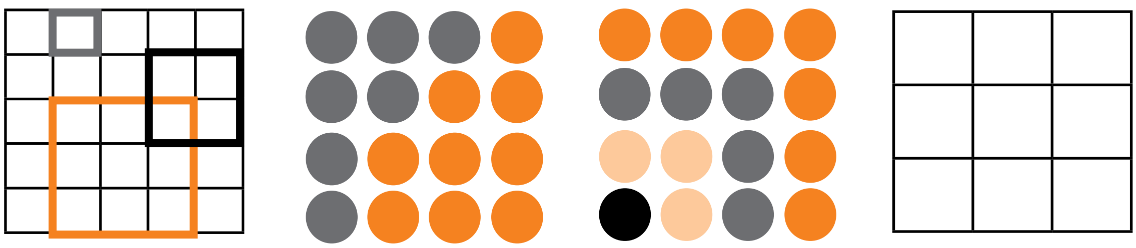
	+1	+3	+5	+7	+9	+11	+13	
0	1	4	9	16	25	36	49	……
0x0	1x1	2x2	3x3	4x4	5x5	6x6	7x7	
	1^2	2^2	3^2	4^2	5^2	6^2	7^2	

Discuss with your partner how you will plan to illustrate this sequence with bottle-tops (or with a diagram) so that pupils will recognise the pattern:



Show that $3^2 + 4^2 = 5^2$

- Which square number is the same as $6^2 + 8^2$?
- How will you encourage your pupils to have a quick mental recall of the square numbers from 1^2 to 15^2?



For more guidance about teaching Indices, check these TDP Lesson Plans:

Year 6 Week 9 Factors, multiples, primes, and indices

Day 3 Indices

Day 4 Squares and square roots

Day 5 Squares and cubes in measuring

Section 6:

Using Brackets and the Order of Operations

When children are introduced to the basic mathematical operations of addition, subtraction, multiplication and division, for the sake of clarity they begin by performing operations on just two numbers:

27 ÷ 3 **23 x 11** **365 + 127** **17.2 – 9.8**

As you have just seen when dealing with indices, multiplying with more than one number doesn't present any great difficulty. You were able to know the value of 10 x 10 x 10 and you could factorise 24 to write 2 x 2 x 2 x 3. Similarly, children will quite happily add several numbers such as 16 + 9 + 20 + 5. They just do the additions or the multiplications one at a time. There is no difficulty because, in each case, the operations are the same and there is no ambiguity.

However, calculations can be confusing when the operations are mixed, such as with

6 + 4 x 12

What is the answer for this calculation?

Did you say 120 or did you say 54?

The answer depends upon what you did first. Did you think "***six*** plus ***four twelves***"

or did you calculate "***ten*** times ***twelve***" ? Either interpretation is possible.

3 x 10 + 15 ÷ 5

How many different answers can you get for calculating this mixed expression?

You could have said that the answer is 9, or 15, or 33, or 39. All of these are possible.

Multiplying first would have simplified the calculation to 30 + 15 ÷ 5 which could be further simplified to 45 ÷ 5 (giving the answer 9) or to 30 + 3 (giving the answer 33).

Adding first would have simplified the calculation to 3 x 25 ÷ 5 which gives an answer of 15.

Dividing first would simplify the calculation to 3 x 10 + 3 and this can be further simplified to 30 + 3 or to 3 x 13 giving alternative answers of 33 or 39.

Pupils will enjoy challenging one-another to find different answers for calculations such as the following which all have mixed operations.

3 x 7 + 5 **13 + 8 x 3** **36 ÷ 4 + 5** **8 + 12 ÷ 4 - 2**

Such an activity will be a useful way to get pupils to practise their number operations in an enjoyable way but, from a mathematical point of view, we should not write calculations which contain ambiguities. To avoid confusion, mathematicians use brackets to show which operation should be done first. In our first example above, the different answers are specified by using brackets

$$(6 + 4) \times 12 \text{ or } 6 + (4 \times 12)$$

In our second example above you would need to write

$$(3 \times 10) + (15 \div 5)$$

if you want to create the answer 33.

As a rule, mathematicians should never write operations which contain mixed operations unless they include brackets to show which parts of the calculation must be done together.

Where will you place brackets to make 99 x 1 + 1 equal to 198?

or to make 6 + 4 x 3 + 2 equal to 20?

Pupils will need practice in writing calculations with brackets to ensure that the intended answer is achieved. You will find examples for this in the Lesson Plans for Year 6.

With some mixed operations, you may need brackets within brackets to ensure that the separate operations are done in the intended order. In our example on the previous page you will see that

$$3 \times 10 + 15 \div 5$$

can have different answers depending on how brackets are introduced:

$$((3 \times 10) + 15) \div 5 \text{ equals } 9$$

$$3 \times ((10 + 15) \div 5) \text{ equals } 15$$

$$3 \times (10 + (15 \div 5)) \text{ equals } 39$$

In case the nested brackets appear confusing, different types of brackets are usually used to show how they have been placed:

$$3 \times [10 + (15 \div 5)] \text{ or } 3 \times \{10 + (15 \div 5)\}$$

Although such complicated-looking expressions are very rare in Primary School mathematics, there is no ambiguity because the use of brackets shows which operations are to be done first and so you will know the correct order of operations which is required.

There may be some circumstances where brackets have not been used in a mixed calculation. Or you may find, for example on the calculator key pad of your phone, that brackets are not available. In this case, to avoid confusion, mathematicians agree that the operations should be done according to the BODMAS rule.

BODMAS is simply the order in which mixed operations should be done:

Brackets (if they are present)
Of (meaning multiply, as in "*½ **of** 16*")
Division
Multiplication
Addition
Subtraction

Check that you can use the BODMAS rule to make 12 ÷ 4 + 5 x 3 – 6 equal to 12.

If you have a calculator or a phone with a calculator app included, test it to see whether it obeys the BODMAS rule or whether it simply does the operations one-at-a-time in the order that you key in each operation. Almost all scientific calculators use the BODMAS rule - but not all of them do so.

Note that some books will refer to the BIDMAS rule. In the BIDMAS rule, the letter "I" stands for "Indices". Because the "Of" operation and the simplification of Indices are both multiplicative operations, it makes no difference whether you follow the BIDMAS rule or the BODMAS rule!

In practice, pupils will rarely need to use the BIDMAS or the BODMAS rule because mixed expressions should always include brackets to indicate the correct order of operations.

Think

"Knowing the order of the operations is highly important in simplifying numerical expressions which have more than one type of operation." **True** or **False**?

"Using brackets is the best way to avoid ambiguity with mixed operations."

True or **False**?

"The BODMAS rule is more important than the BIDMAS rule."

True or **False**?

Watch

Watch the video clip MM8V6 on your phone.

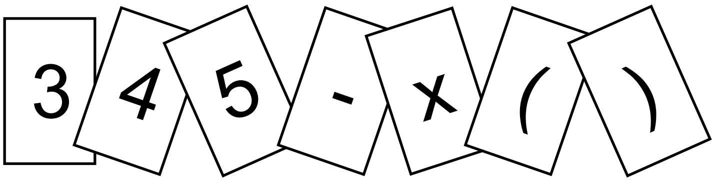
As you watch the lesson extract, think about how the teacher engages the pupils in the lesson.

Reflect

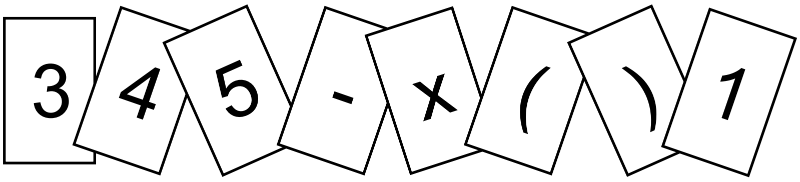
In the video, you saw pupils having a group competition to create a context for learning mathematics.

- How do you know that the pupils enjoyed this practical activity?
- How did their enjoyment contribute to the learning of the mathematics?

Work with your partner in school



Using all these five cards, what is the smallest number you can make?
What is the largest number you can make?



Add a card with the number 1. Can you arrange all the cards to make 36?
Can you use the same cards to make 32?

Discuss with your partner how you could you use a similar set of cards to give pupils some practice in using brackets. *If you use a division sign, make sure you include the possibility of making whole number divisions.*

Check the TDP Lesson Plans for P6 Week 21 ***Order of operations*** for more detail and support for teaching about BODMAS and using brackets.

Summary of Module 8

This module focused on the writing of the basic number operations appropriate for Primary Years 4 – 6. You saw that pupils need to know how to use columns labelled “H T U t and h” in order to ensure that their concepts of place value support their growing ability to calculate with larger numbers and with decimal numbers. The common practice of *“moving the decimal point”* when multiplying and dividing by multiples of 10 was shown to be the result of place value changes to the digits in a number.

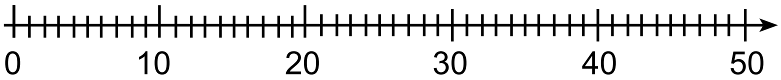
The module also included several references to developing pupils’ mental skills. You saw that the ability to recall single-digit multiplication facts is a key skill, not just for multiplication but also for division. The module suggested that multiplying by 10 and by 11 should be thought of as single-digit operations. The 10x and 11x tables are the easiest for pupils to remember and so using these as single-digit multiplications instead of 2-digit multiplications can avoid unnecessarily long calculations. You recognised that the mental ability to estimate an answer for a calculation or for a problem plays a useful part of our daily lives as well as in mathematics. Using the number line from the “Ideas …” section below will assist mental skills as well as developing pupils’ conceptual understanding of the four basic number operations.

The final sections of the module explained the mathematician’s use of Indices and of Brackets.

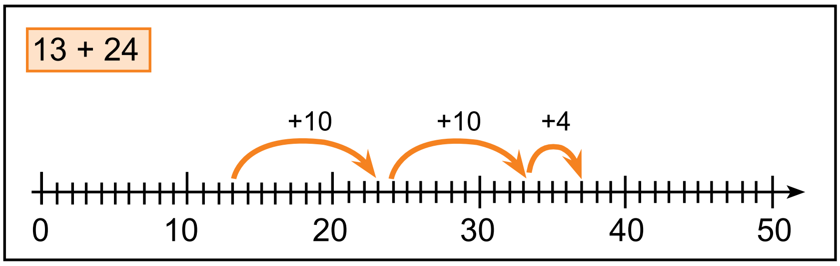
In each of the lesson extracts which you watched on your phone, you saw teachers trying to engage their pupils with clear explanations of the mathematical steps. These teachers had often prepared exercises for pupils to practice independently in groups so that the pupils could become owners of the new skills. You were encouraged to help your pupils to understand the mathematical operations rather than to just blindly copy routines. Teaching aids were rarely available and you saw teachers resorting to “chalk and talk” but the “Work with your Partner” sections have suggested activities which will encourage more pupil activity. We know that less teacher talk and more pupil activity (sometimes working alone, sometimes in pairs and sometimes in groups) create better learning.

Module 8 - Ideas to try in the classroom

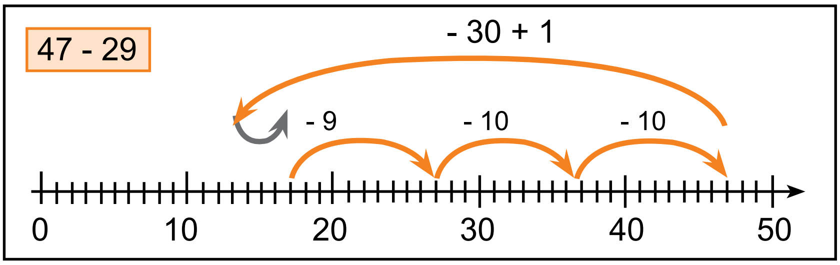
Paint a number line on a wall in your classroom: from 0 to 50 is a good range to have.



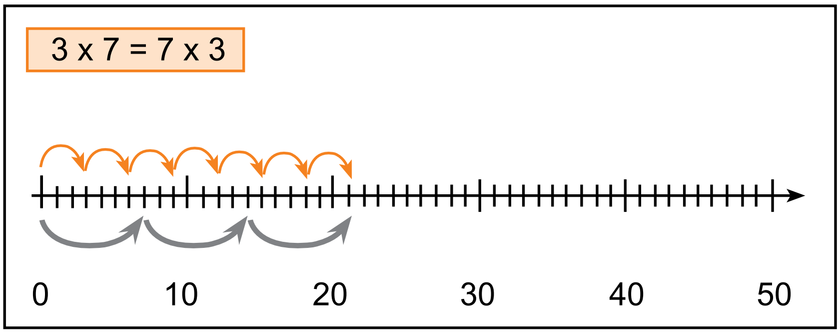
Use it for teaching about addition, for example …



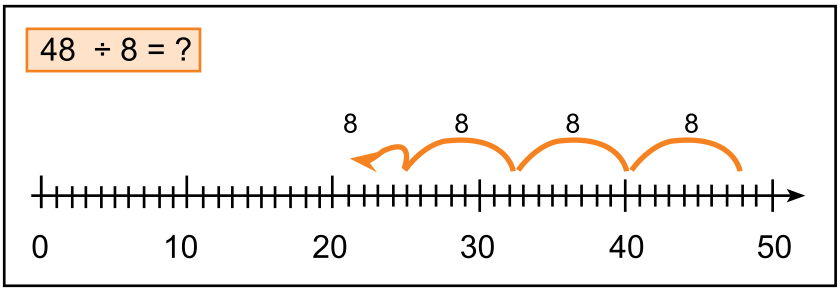
… or exploring ways to subtract, fpr example …



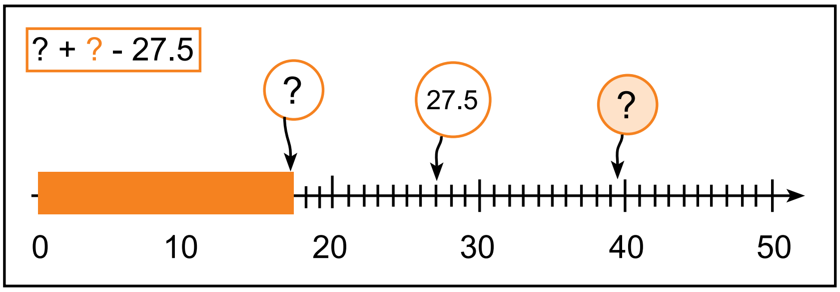
Use it for multiplications …

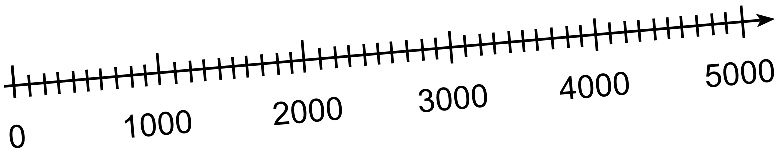


… divisions …

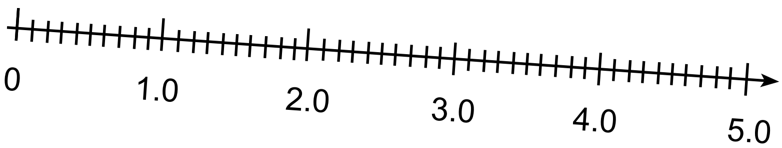


… and for estimations.





You can also change your line by using chalk to add some zeros, or a decimal point, when you need to.



As you know, the module is designed to help you to develop your teaching skills. Experiencing change in your classroom is an important part of this professional development. To ensure effective changes and to build on these, you will need to

- try out new ideas and be perceptive about what contributes to better learning;
- notice what pupils like and notice what strategies or techniques work well.

When you want children to do more than just repeating your examples,

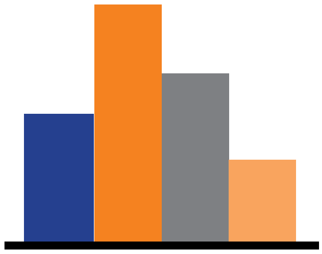
- help them to engage with the mathematical ideas which underpin the work;
- challenge their thinking so that they develop understanding; and
- notice what enables you to be successful in helping pupils to work independently.

Remember that when you try an activity that you have not used before, this will also be new in style to the pupils. They may not feel confident to respond in the ways that you had hoped. It might take several attempts for them to recognise that it is their own thoughtful enquiry about the mathematical idea that is more beneficial than passive and repetitive rote learning.

Make a note here of any topic, question or comment on this module that you want to discuss or share with your Teacher Facilitator or colleagues at the next Cluster Meeting:

Module 9:
Everyday Statistics

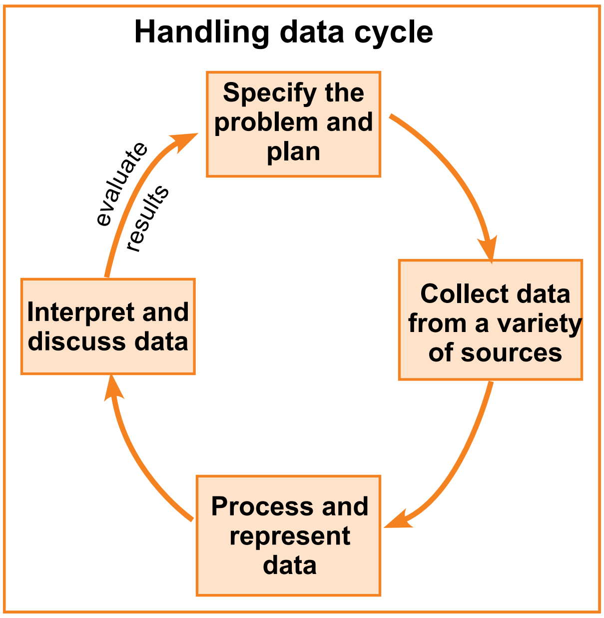
Module 9:
Everyday Statistics



Statistics is the branch of mathematics which studies how to collect, organise, analyse and interpret data. Data (the plural of datum) is the mathematician's word for a collection of observations, measurements or facts. The handling of data is always done in response to a question:

How many people live in Nigeria? Is the population increasing or decreasing?
How many ballot papers are needed for the next election?
How many schools are needed in our State?
If I sell shoes, what sizes do I need to have on my stall? Do I need more of some sizes?
Would it be more efficient to have a Health Centre in each village or a Hospital in the town?

The answers to these, and to many more questions, enable us to cope with the practical demands of everyday life and the functioning of our country.



Handling the data requires four different skills:

- first of all, you need to know what question needs to be answered and what information is needed to answer that question;
- secondly, you need to collect the relevant information; then
- the data needs to be organised and displayed;
- finally, the information needs to be interpreted. *What does the data tell you?*

When the results have been evaluated, it is often the case in Statistics that the cycle needs to be repeated because the interpretation of the results may lead to more questions or to the recognition that more detailed information is needed.

The ability to investigate, gather, present and interpret information will help pupils to think logically and to reason carefully. So this module is designed to guide the teacher to introduce these new skills in an effective way.

In Years 1 – 3 pupils will have collected data about themselves: their heights, their birthdays or their ages. In Years 4 and 5 they will learn how to record this information in a Bar Chart and to use a Tally Chart to collect other information. Pupils will learn how a Pictogram can be used to show both small and large quantities. Year 6 pupils will begin to interpret data, using their bar graphs and pictograms to evaluate the information and using the mode, median or mean to suggest an average figure for their data.

Objectives of the module

By the end of this Module, teachers will be able to support pupils to:

- prepare and draw a tally table; (Year 4)
- prepare, draw and interpret a pictogram; (Years 4 and 5)
- prepare, draw and interpret bar graphs and tables; (Years 4, 5 and 6)
- identify the mode and median values for groups of data; (Years 4 and 5)
- calculate the mean average of given data.. (Years 5 and 6).

.

Section 1:
Tally Table

A table of tally marks is used to collect information ***as it happens***.

For example, a local government office wants to improve a road junction where many accidents have occurred. A survey is to be made of the number of cars, lorries, bikes and motorbikes, as well as pedestrians, which use the junction. The local officer records the information: each time a vehicle or a person crosses the junction, he makes a tally mark on his list.

To make it easier to count the tally marks, the official marks them in fives: *one mark is made for each of the first four vehicles; the fifth tally is drawn across the other four.*

Vehicles turning left ←	Vehicles going straight across	Vehicles dropping passengers	Vehicles turning right →
卌 卌 ||	卌 卌 卌 ||||	卌 || 卌	||
Passengers crossing ←	Pedestrians crossing ↑	Pedestrians crossing ↓	Pedestrians crossing →
卌 ||	卌	卌 |||	||||

You can arrange a similar tally for your class. You will need a list of fruits that pupils like to eat.

Line up all the pupils outside the classroom before the lesson starts.

As the pupils enter the classroom, one by one, they will tell you their favourite fruit. As they do so, make a tally mark against their choice.

Orange
Mango
Banana
Pineapple
Watermelon
Pawpaw

Fruit	Tally Marks	Total
Orange	𝍸 𝍸 ||	
Mango	𝍸 𝍸 𝍸 |	
Banana	𝍸 ||	
Pineapple	𝍸 𝍸 |	
Watermelon	𝍸 𝍸 𝍸 𝍸 |||	
Pawpaw	𝍸 ||	

When all the pupils have declared their favourite fruit and have sat down, explain to the pupils how you have made the tally table using bundles of five tallies. Let the pupils copy your chart.

Now ask them to count the number of tallies recorded for each fruit.

The pupils will see that the bundles of five make the counting easy to do: "*Five, ten, fifteen*, …"

Write the total for each category in the third column.

Your tally table provides an answer to the question "Which fruit is the most popular in this class?"

> Note that another way to collect that same information could be to wait until the whole class is seated; then ask for pupils to raise their hand when you call out the name of each fruit. You would then just count the number of children choosing each fruit and write only the number in your list.
>
> **This is not the same thing as making a tally table.**
>
> You would not need to make a tally table if you can just count the number of things in a category or if you already know the totals.
>
> A tally table is used only when you collect information about something over time, recording each event ***one by one***.

A headteacher is helping his Year 4 teacher to mark the Mathematics test for the Year 4 pupils. Because they want to know how many pupils are doing well, they make a tally table to categorise each pupil's score.

	Tally	Total
Excellent (more than 80%)	//	
Good (from 65% to 80%)	卌 ////	
Satisfactory (from 50% to 65%)	卌 //	
Below half (less than 50%)	////	

The Headteacher makes a tally mark after each test is marked, one by one. In this way he records whether each test mark is Excellent, Good, Satisfactory or Below the 50% midpoint. When they have finished marking the tallies, he will find the total for each group.
Can you tell from the table how many pupils' tests they have marked so far?

If your school is close to a main road, pupils could stand inside the school gates and record the colour or the type of vehicles that pass the school. This will give them the experience of making a tally table.

Colour of cars that pass the school	Tally	Total
black	|||	
grey	𝍸 𝍸 |||	
blue	𝍸 ||	
red	|||	
white	𝍸 𝍸	

Vehicles that pass the school	Tally	Total
lorry	𝍸 𝍸 ||	
pick-up truck	𝍸 𝍸 |||	
car	𝍸 𝍸 𝍸	
motorbike	𝍸 |||	
bicycle	𝍸 𝍸	

If this is not possible, place a mixture of ten coloured stones (all the same size) in an opaque bag. (You could use ten sweets with different coloured wrappers.) As you pass around the class, each pupil in turn takes a stone without looking into the bag. When the stone is taken out, the pupil calls out the colour of the stone and then returns it to the bag. A tally mark is made to show the colour of each stone selected. After every child has selected a stone, the pupils count the total of tally marks recorded for each colour.

Colours picked at random	Tally	Total
white	𝍸 𝍸 ||	
red	𝍸 𝍸 |||	
yellow	𝍸 𝍸	
blue		
black	𝍸 |||| ||	
green	||||	

Tell the class that there were 10 stones in the bag (the stones need to be a mixture of at least three colours in different amounts. For example, 5 reds, 2 whites, 1 black and 2 yellow.) It will be interesting to see if the children can use the totals in their tally table to answer the questions:

- *What colours do I have in my bag?*
- *How many stones of each colour do you think I have in my bag?*
  Do you think that their tally table will allow them to make a good prediction of the actual number of each colour?

Think

Which of these statements are. **True?**

1. Tallies are recorded as a survey is made. **Yes / No**
2. Each tally mark represents one person or thing. **Yes / No**
3. Tallies are grouped in 10s. **Yes / No**
4. The fifth tally cancels out the previous four tallies. **Yes / No**

Watch

Watch the video clip MM9 V1X on your phone.
You will see the teacher and her class making the survey of pupils' favourite fruits described on the first page of this section.

Reflect

Compare the children's activity in this video clip with the suggested activities to carry out a vehicle survey and to pick coloured stones from a bag. Which activity, do you think, will give children the best experience of making a tally table? Why?

Work with your partner in school

One of you chooses a P4 child's reading book and the other chooses an adult reading book or, perhaps, this Teacher Guide.

Each of you chooses a short paragraph from your book – one with about 50 words will be a good size.

Make a tally table for the number of letters in each word of your selection. Just select the words one at a time, count its letters, and mark a tally in your table.

Children's Book

Number of letters in word sample	Tally	Total
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Adult's Book

Number of letters in word sample	Tally	Total
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Compare your tally table totals. Do your tables suggest that adult books have longer words than children's books?

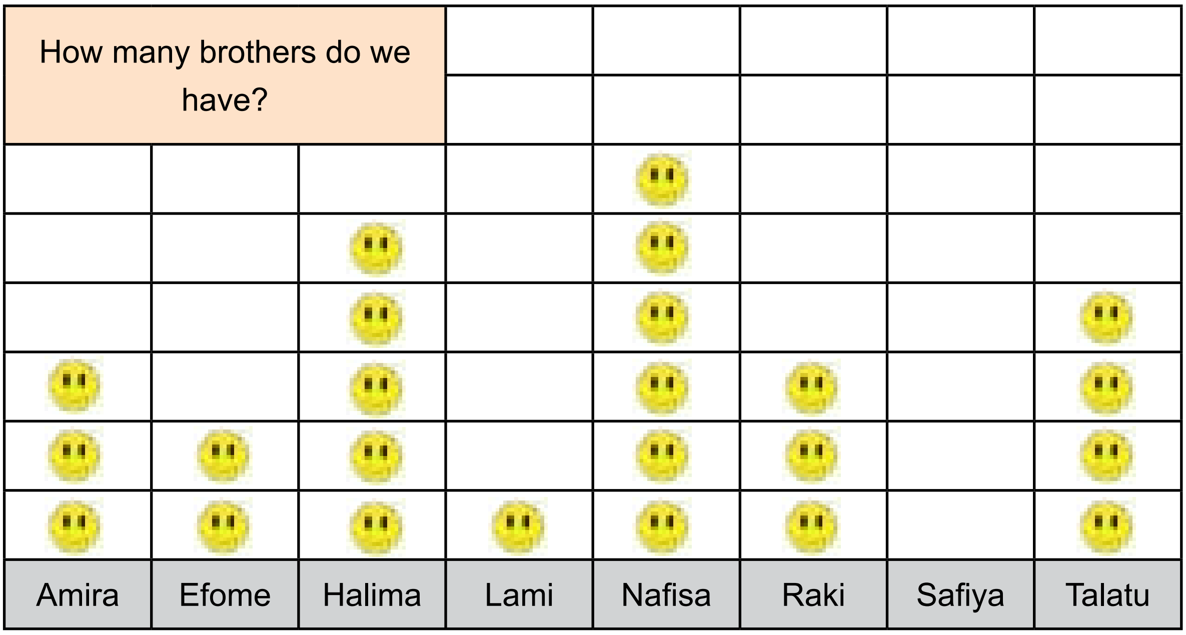
You could ask your pupils to carry out a similar survey to answer the question: *Do children's books have shorter words than adult books?*
(You will need at least one pair of books for each group of children.)

Section 2: Pictographs

Pupils' first experience of recording statistics is likely to be in Year 3 when they place a bottle-top to show a category they choose - like the pupil in the photograph below who is placing a counter to show that her favourite colour is white. The teacher draws a grid on paper, or on the floor with chalk, and the pupils each place a counter, a stone, a bottle-top or a seed in the column of their choice. The pupils actually create a physical graph with the concrete materials.

This concrete experience is next translated into drawing a chart on the chalk-board. Now, the teacher labels columns on the board and pupils show the category to which they belong by marking a cross in one column. The charts created in this way will probably show some information about the pupils themselves: their favourite food or the month of their birthday, for example.

When children make a Statistics chart of their own in Year 4, they will still show information about themselves because this is the data that they already know and can use immediately.



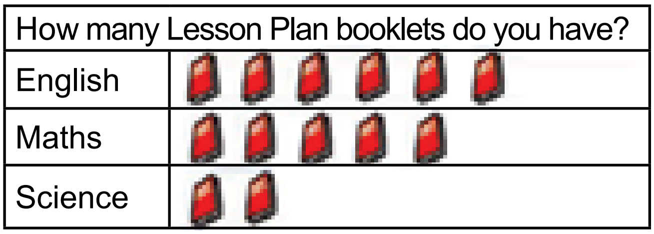
Instead of drawing crosses, the eight girls in this group have put faces to show the number of brothers they have. One face represents one boy. (Each group of boys in this class did a similar chart to show the number of sisters they have. Their faces each represented one girl.)

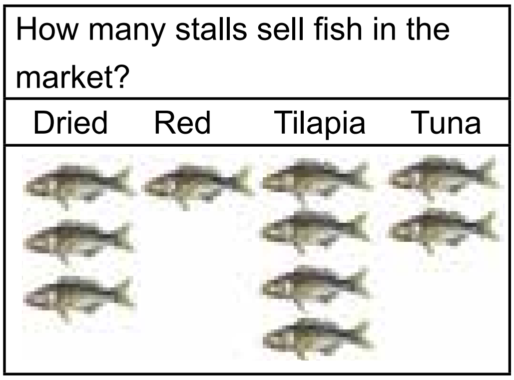
The faces used in the chart are “pictograms”. A pictogram could be any symbol, or you could just use a square block to represent each person. Mathematicians prefer to use a symbol that suggests or illustrates the idea that it represents – in this case, a person. The whole chart is usually called a pictograph.

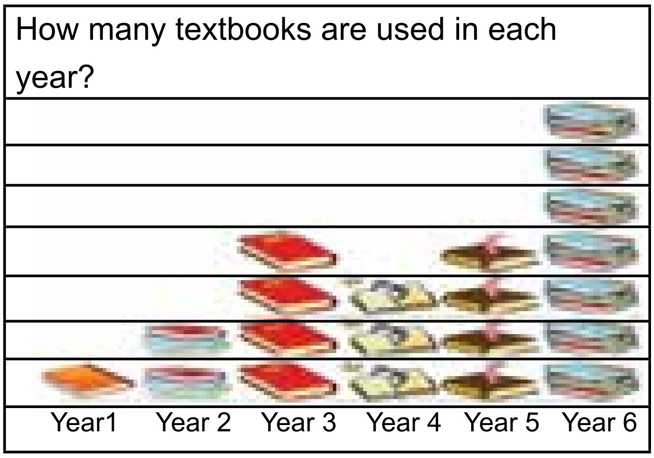
This example shows how effective a pictograph is: the data is presented visually. The pictograms tell you that the chart is about people. You can see that Nafisa has 6 brothers, just by looking at the grid. You can see that Safiya does not have any brothers - the pictograph makes it obvious.

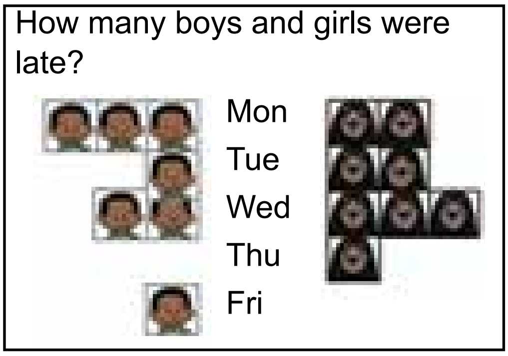
Making pictographs like this develops into the bar charts and graphs that pupils will learn about in Years 4, 5 and 6. This development will be the focus of Section 3. Here, in this Section 2, you will find advice on how to support pupils with the more sophisticated ideas associated with using pictograms for larger quantities.

You will notice that pictographs are drawn in a variety of orientations: left, right, up or down…

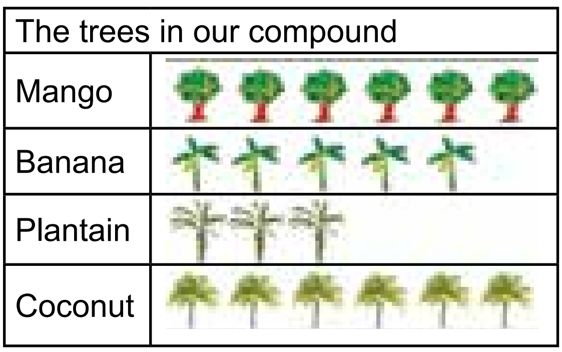


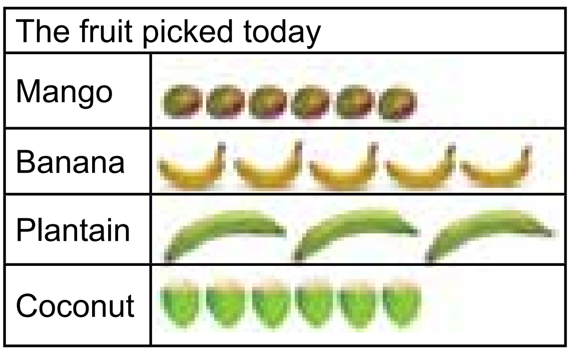






… but the information in each of these examples is clear because each pictogram represents one book, or one fish stall, or one boy or one girl, even when there are different images in the same chart. Each pictogram conveys its meaning through its pictorial resemblance to the physical object that it is describing. However, if the pictograms are of different sizes, a pictograph can be misleading.





Compare these two pictographs.

In both charts, different pictograms are used to show each type of tree and each type of fruit. Although the tree pictograms in each category are different, the individual pictures are all the same size. So the visual impression of the pictograph still shows that the coconut palm trees are the same number as the mango trees, You can immediately see that the number of plantain trees is only half the number of mango trees. However, the fruit pictograms are different sizes and so they give a misleading impression.

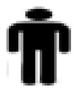
It appears that the plantains were the biggest group but, in fact, only 3 plantain fingers were picked, compared with 6 coconuts and 6 mangoes. Because the pictograph is designed to give its data visually, the second fruit graph is not correct. It gives the impression that bananas and plaintain fingers are both more than the mangoes or coconuts. To make the pictograph acceptable, and visually informative, the pictogram symbols should all be the same size so that the numbers in each group can be compared.

Young pupils are not very good at drawing their pictograms all the same size so it is often helpful for them to draw around a bottle-top or a *Maggi* stock cube when they draw their own pictographs.

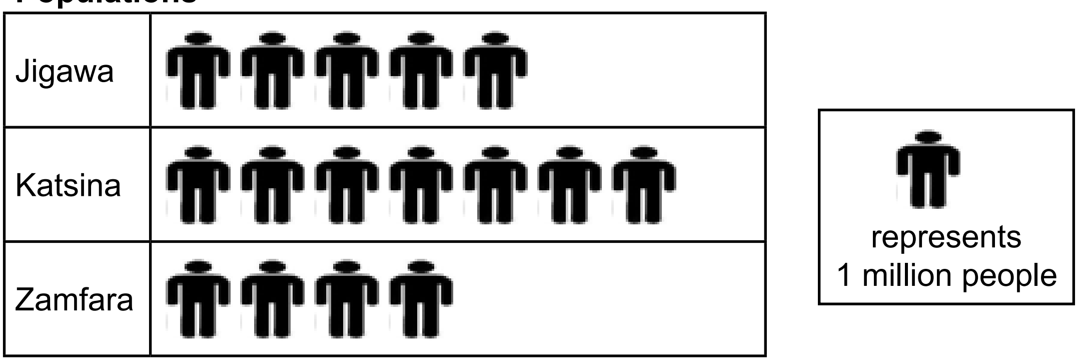
"How many people live in the States of Jigawa, Katsina and Zamfara?"

When you draw a pictograph to compare the number of people living in these three States, you would need to draw millions of pictograms if you were to use one symbol for each person.

That would be impossible! So to draw such a chart, you will need to use one symbol to represent many people. To show these large numbers, the chart below uses this pictogram to represent one million people. →

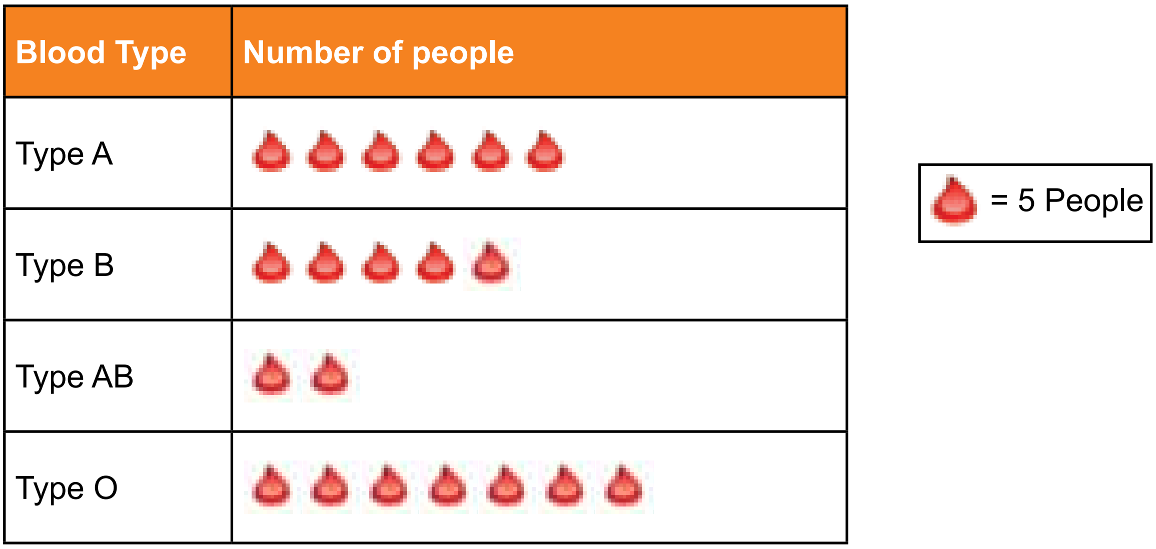


Populations

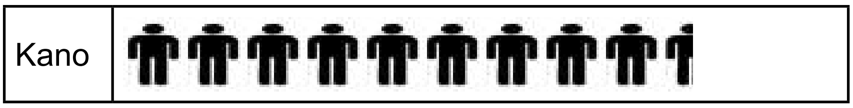


You will understand that, when a pictogram stands for more than one person or object, the chart needs to include a key so that the reader will know what the symbol represents.

In the pictogram on the right, which shows the blood groups of 100 people visiting the local hospital, the symbol of a drop of blood is used to represent 5 people. Pupils will need to use their mental recall of the 5x multiplication table to be able to read this data.

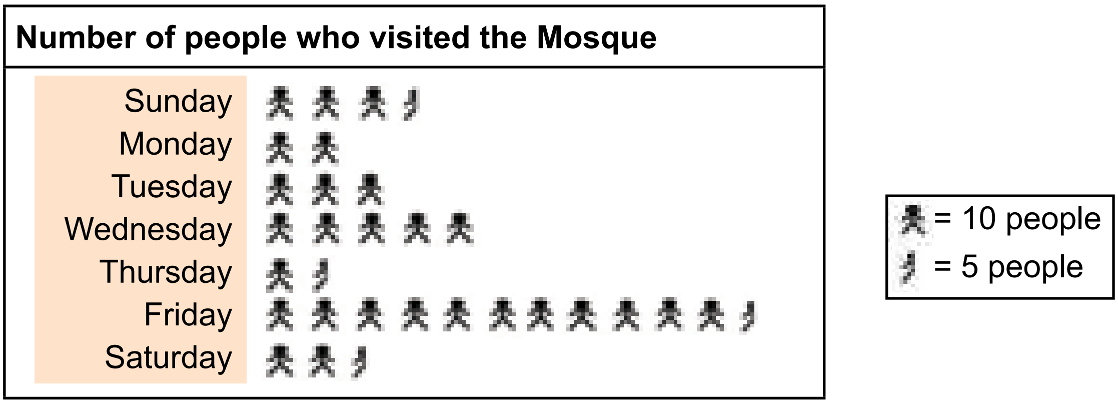


When a pictogram is used to represent a number larger than one, you may be able to use only a fraction of it to give more accurate information. Looking at the populations of the three states above, the data has been rounded to the nearest million. If you want to include the population of Kano State in the comparisons above, you could add the following information:

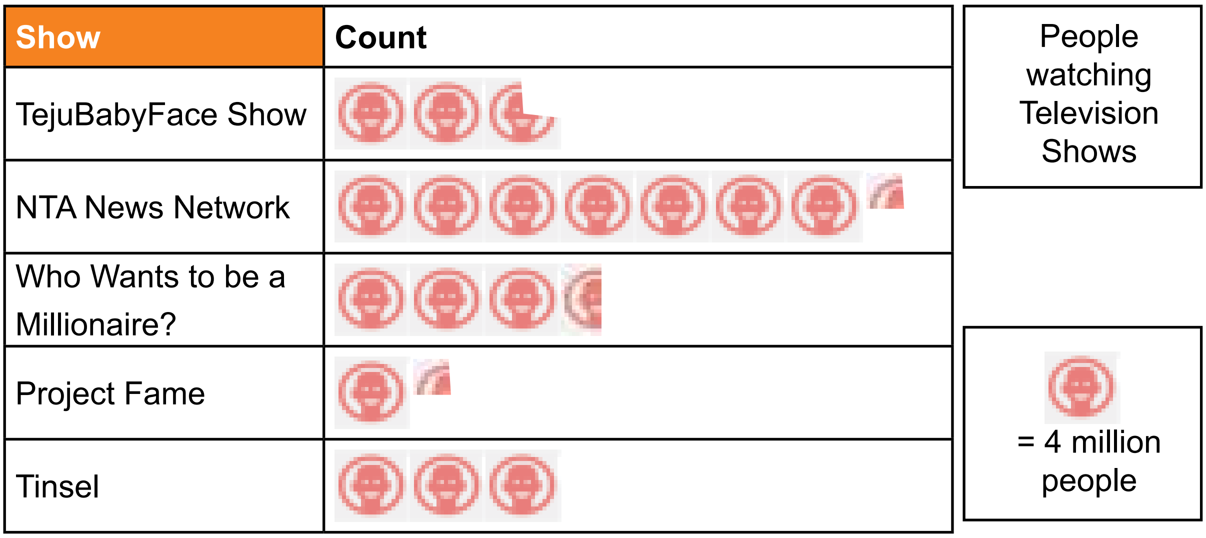


What is the population of Kano shown by these pictograms? The pictogram of half a figure represents ½ million. So the population of Kano is 9½ million people, or 9,500,000.

The pictograph on the right shows the number of people who visit a village mosque during one week. The stick-man symbol represents 10 people. So you can see that 25 people visited the mosque on Saturday. But what if the number of people who visited the mosque on Sunday had been 37? The pictogram used does not allow this



number to be shown accurately. Using such a symbol is fine to show data which has been rounded to the nearest five but it does not allow greater detail. If you want to use a pictogram which represents many objects but which also allows more detail, you would need to choose a symbol which can be divided into fractions and still remain readable. The following pictograph is one example because a circular symbol is used to represent 4 million people who watched the television shows.



This allows the pictograph to show that 5 million people watched the *Project Fame West Africa* show because the quarter symbol shows 1 million.

As you have seen, the pictogram a mathematician chooses for displaying information about large numbers will usually have a visual link to the data. It will also be influenced by whether there is a need to show only a fraction of the symbol. The final example below, showing letters posted from an office, is a good choice because the symbol of an envelope conveys the idea of a letter and its rectangular shape makes it easy to show some detail of the data. For example, the data in the table on the left can be illustrated in the pictograph on the right.

Day	Letters sent
Monday	10
Tuesday	17
Wednesday	29
Thursday	41
Friday	18

Day	Letters sent
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	

 = 10 letters

Think

Why do you think that mathematicians use pictograms to display data?
Have you used a pictograph before with your class?

Watch

Watch the video clip MM9V2 on your phone.
You will see the teacher helping her class to “read” a pictograph.

Reflect

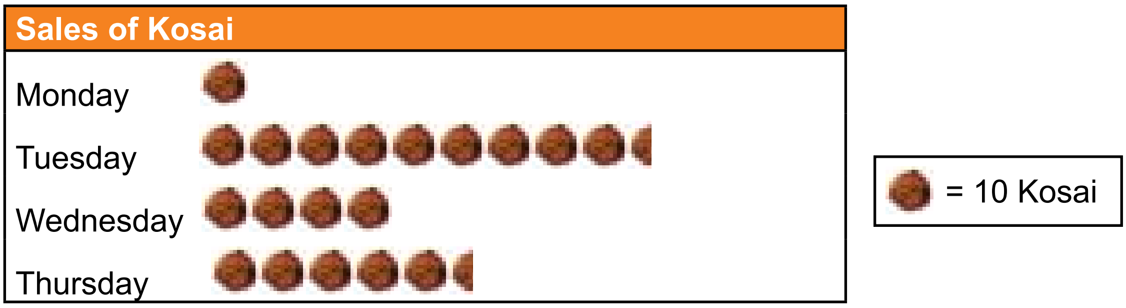
1. What are the similarities and differences between a tally table and a pictograph?

Similarities	Differences

2 Ibrahim made a survey of the 60 people in his neighbourhood. He enquired how many of them go to the mosque on different days of the week. To represent the data, would you recommend him to use a tally table or a pictogram? Explain your answer.

Work with your partner in school

1. Amira’s mother fries fresh kosai and sells them in the market.



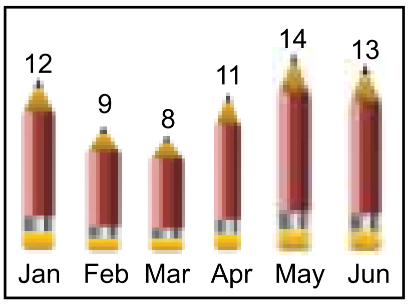
Plan a lesson to make use of this pictograph.
What prior knowledge will pupils need to have before you teach them using this pictograph?

What information can you ask pupils to find from this pictograph?

2. Tell your partner at least two reasons why you would **not** use this pictograph in your teaching:

3. In what way is this pencil chart a pictograph? In what way is it not a pictograph?

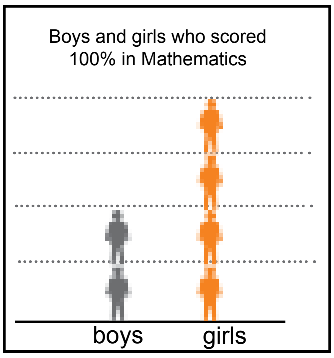
Pencils used in 6 months

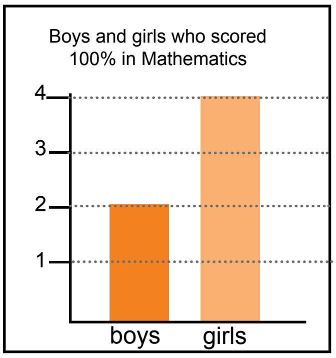


4. Check the Lesson Plans for more guidance on teaching about pictographs:
   - P4 week 18 Statistics Day 2 Pictograms
   - Day 3 Late for school
   - P5 week 8 Statistics Day 1 Pictogram
   - Day 2 Making Pictograms

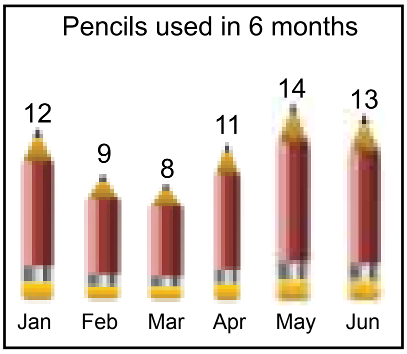
Section 3:
Bar Graphs

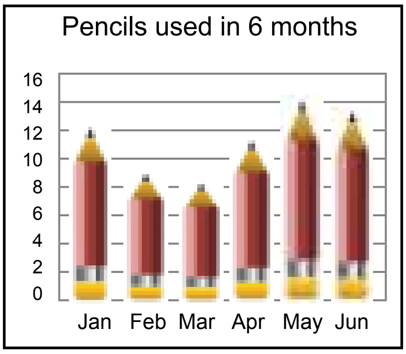
Bar graphs can be simple, showing data about just one thing - or very sophisticated, showing several pieces of information at the same time. In this unit you will be guided on the teaching of Bar Graphs appropriate for Years 4 through to Year 6. You will see that displaying data using bar graphs follows on naturally from the simple pictographs where one symbol was used to represent one object.

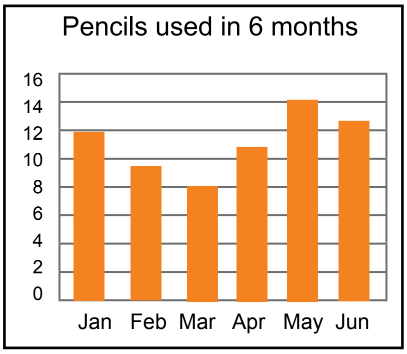




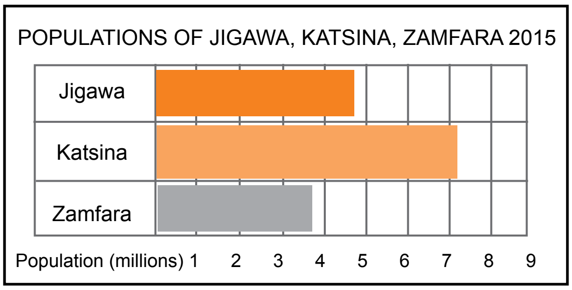
Because the bar graph only uses rectangles drawn to the necessary size, the bars can only show the number in their group by using a frequency scale at the side of the diagram. You knew there were 2 boys in the pictograph above because each symbol represents one pupil. Without the scale on the left-hand axis of the bar graph, you would not know how many boys or how many girls were represented by each of the bars.

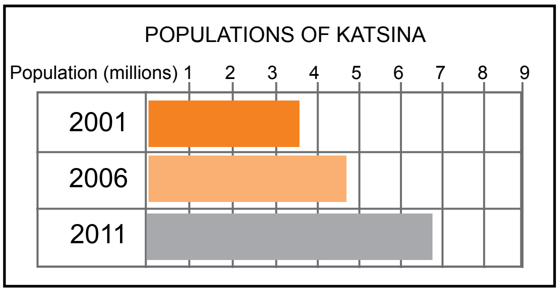






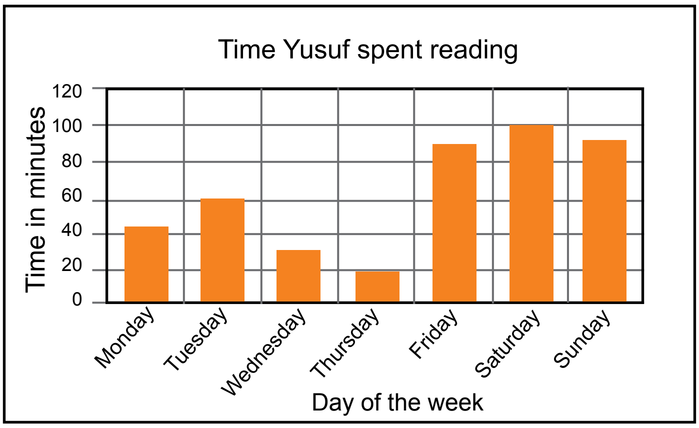
When you looked at the pencil graph at the end of Unit 2, you saw an unusual pictograph where the lengths of the pencils matched their numbers. This is quite similar to a bar graph but it needed the numbers to be written above the pencils because, otherwise, the chart would have looked crowded like the one in the middle. You can see why mathematicians like to use a bar graph like the one on the right – it is clear and unambiguous.





The essence of a bar graph is that it provides a visual display for comparing quantities. The heights or lengths of the bars depends upon the number in the category or the frequency of that item. The bars may be used to compare several items at the same time or to compare something as it changes over time. Reading between the lines can give quite accurate information.

In year 4, the emphasis of your teaching about bar graphs will be to help pupils to read and to interpret bar graphs with simple data.

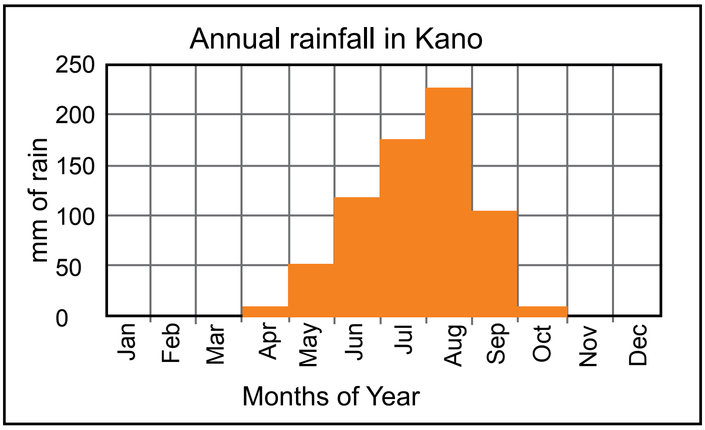


Pupils should learn how to solve comparison and sum-and-difference problems using information presented in bar charts like this.

- What do the words across the bottom of the graph tell you?
- What do the numbers at the side of the graph tell you?
- On which day did Yusuf spend most time (or least time) reading?
- On which day did Yusuf spent an hour-and-a-half reading?
- How much time did Yusuf spend reading on Wednesday?
- How much more time did he spend reading on Tuesday than on Monday?

Through such an activity, pupils will learn that a bar graph needs:

- **a title** to tell the reader what the graph is about (e.g. “Time Yusuf spent Reading”);
- **grid lines** to create the scale (choose the number lines to match the data);
- **labels** for the data displayed (like in the first graph on the next page);
- **categories** for each bar of data ((like in the first graph on the next page);
- **bars** to show the data.



- What is the title of this graph?
- What does annual mean? How is this connected with the names of the columns?
- What do the numbers on the side axis tell you?
- Which is the wettest month in Kano?
- Which two months have the same amount of rain?
- How many months have no rain?
- How many millimetres of rain did Kano get in June?

Pupils will enjoy an activity where each member of their group, in turn, asks one question that the others group members have to answer. Of course, to do this, they will need a bar graph to talk about, so look out for suitable graphs in books and newspapers. Make a collection of useful bar graphs to add to your teaching tools.

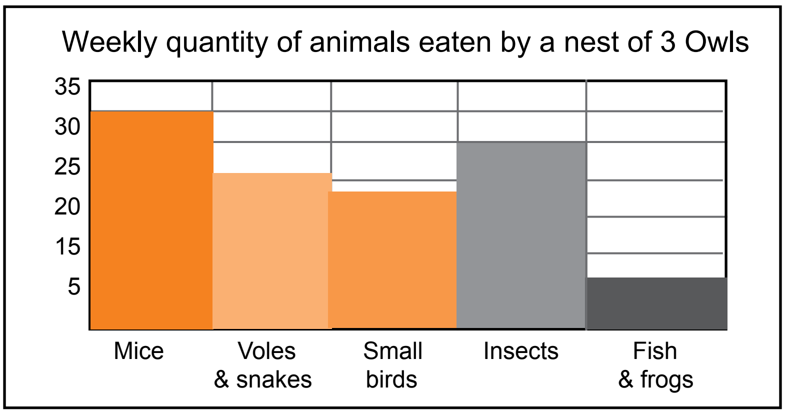
In Year 5 the emphasis changes to pupils drawing their own bar charts. So pupils will need some data to display. For example, they could draw a bar graph to show how many pupils were late last week.

Monday	Tuesday	Wednesday	Thursday	Friday
9	8	5	6	8

To draw their own graphs, the pupils will need paper with a square or rectangular grid. Otherwise, their task of drawing the columns will be challenging and time-consuming. Nevertheless, it will probably be the choice of a suitable frequency scale that Year 5 pupils will find to be the most difficult part. For this reason, you should initially choose simple data (like the number of late pupils) which can be shown with a simple scale such as 1, 2, 3, 4, … or 2, 4, 6, 8, … or 10, 20, 30, 40, …

The frequency axis, which displays the numbers to show the size of the bars, is most often on the left-hand side, with the bars rising up from the bottom. But, as you have seen with the population bar graphs for Jigawa, Katsina and Zamfara, the frequency axis showing the measurement could be at the bottom or at the top, with the bars extending towards the right. Whether the frequency axis is on the left or on the bottom or at the top of the graph is not important. What is important is that pupils learn to label the **lines** of the frequency axis, **not the spaces between**. If the lines are labelled, pupils can judge where the bars will reach, whatever their measurement.

Pupils will happily tell you their favourite colour, their favourite fruit or their favourite meal so they can draw a bar chart to display this information. However, because this data is so easily available, take care not to use the same data each time you teach about tally tables, pictograms and bar graphs. Keep a look out for other interesting information which pupils will be stimulated by. For example, here is a bar graph about the food which owls catch and eat.



Because owls are nocturnal, pupils may not know much about the four or five different types of owl found in northern Nigeria and the food that these owls eat. You can ask many questions about the information shown in the graph.

- What food is the most frequently caught by owls?
- What food do owls like but you think might be hard for them to find?
- How many small mammals, like mice, did the owls catch in this survey?
- How many small birds did they eat?
- Do you think that the owls got more food from insects or from small birds?

These questions will help pupils to read and infer data from the graph. What other data can you find which will interest your pupils and, at the same time, help them to draw bar graphs?

Groups of Year 6 pupils will construct their own graphs from information which they collect themselves. It will be necessary for you to help them decide what question they want to investigate so that they are realistic about the data they can collect. When they have gathered sufficient data, pupils should design and draw a bar graph to present their information.

Finally, they will discuss and interpret the data that their graph illustrates. Ideally, they can report a brief summary of their findings to the rest of the class. In this way, they will complete the data handling cycle which was described in the opening section of this Module.

Pupils should be encouraged to have their own questions to investigate but you might suggest some possibilities to them:

- What activities do children in the class do after school?
- What sport do pupils like to play or to watch?
- How many animals do pupils have at their home?
- What different crops do families grow in their farm?
- When you throw a dice to play a game, how many times do you score 1, 2, 3, 4, 5 or 6?
- Do teachers prefer different food to pupils?

The graphs that pupils will draw as a result of asking these questions will show ***discrete data,*** like all the other bar graphs we have considered so far. Discrete data has separate categories, with each category being distinctly different. For example, each item of owl's food will fit into only one of the five categories. A score on a dice can only be one of six different numbers. Similarly, answers and information about food will fall into discrete categories, such as favourite types of Tuwo that teachers prefer.

Tuwon Shinkafa	Tuwon Dawa	Tuwon Masara	Tuwon Acha	Brabusco
9	6	5	2	8

Other questions may result in producing data that needs to be grouped.

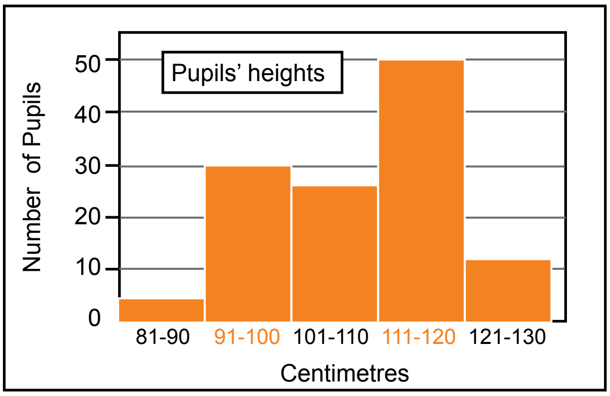
- How tall are the pupils in Year 6?
- What marks did pupils get in a test?
- How many minutes does it take pupils in the class to walk to school?

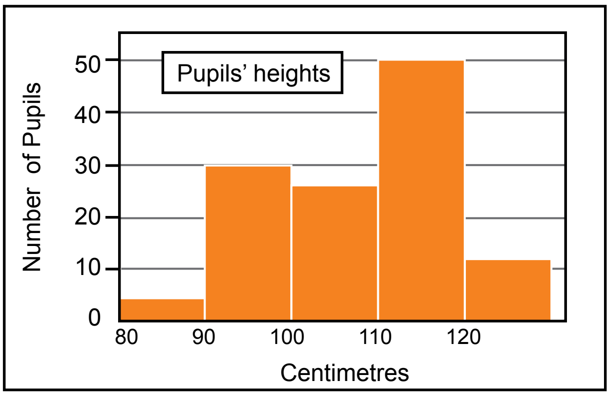
Because the heights of pupils, for example, (or marks, or times) are from a ***continuous*** list of measurements, the data needs to be grouped. Otherwise, pupils might have very many measurement categories all with only 1 person in the category and none in others.

Height of pupils	81-90cm	91-100cm	101-110cm	111-120cm	121-130cm
No. of pupils	4	30	25	50	11

A bar chart for ***continuous data***, like these heights of pupils, is drawn in the same way as for discrete data but there should be no gaps between the sections so that everyone's height can be included in one category or another (like in the graph below on the left).

For continuous data, the categories at the bottom could also be written as a continuous numbered axis (like in the graph below on the right). In such an example, you will need to know into which category a boundary measurement like 90cm falls.





Note that each category of the grouped data is the same size. The data has been collected in groups at 10cm intervals. This ensures that the bars of the graph are all the same width. Pupils should recognise that a boundary measurement like 90cm should not appear in both groups 1 and 2. Note that the first category is 81-90cm and so includes 90cm but it does not include 80cm. In which group would you place someone whose height is 90.4cm?

Check that children can answer questions about data presented in different ways:

- *Are they able to make connections when looking at the same data presented differently in Tally Tables, Pictographs or Bar Graphs?*
- *Can they answer questions about the data using inference and deduction or only direct retrieval ?*
- *Are they able to present data in different ways?*
- *Do they label axes correctly?*
- *Do they understand the scale and do they use an appropriate scale when presenting data?*
- *Do they know how to group data when it is from a continuous measurement source, not in discrete categories ?*

If, as a Year 6 teacher, you can answer "Yes" to each of these questions at the end of the year, you will have done a very good job in teaching children about Statistics and the necessary skills to handle data.

Think

Check that you, yourself, have answered each of the questions in orange included in this unit.

Watch

Watch the video clip MM9V3 on your phone. As you watch the two episodes from a lesson on Bar Graphs, make a note of the materials that this lesson needed.

Reflect

The children in this lesson succeeded in drawing their bar graphs using paper without a square grid. Were they still successful? What equipment helped them to draw their graphs?

Fatima gave an explanation of how her group had drawn the bar chart to show the number of pupils in each year of the school. But she did not explain how her group had decided what the scale of their frequency axis should be. Drawing the bars is an easy task for the pupils but choosing an appropriate scale is more demanding. How would you have intervened to make the decision-making about the scale be more explicit? *Steps of 10 had been suggested by Ibrahim but could pupils have used steps of 5 or 20? Which would be more efficient?*

Practice

Using the data on the previous page, draw a bar graph to show the favourite type of Tuwo that the teachers in this sample preferred. *What scale will you use for the frequency axis?*

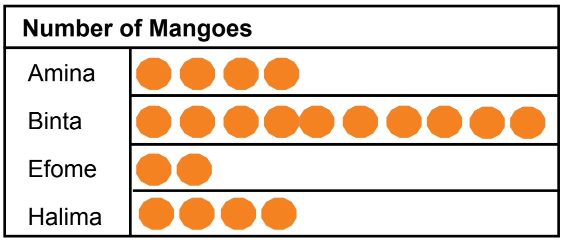
Work with your partner in school

Together with your partner in school, plan a lesson appropriate for your year to teach bar graphs to your pupils. Will you focus on interpreting a graph, drawing a graph or answering a question? The following Lesson Plans may be helpful:

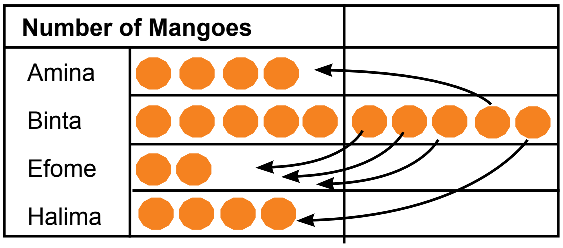
- P5 week 8 — Statistics — Day 3 Vertical bar graphs
  Day 5 Reading bar graphs
- P5 week 18 — Statistics — Day 4 A bar chart
  Day 5 Absent from school

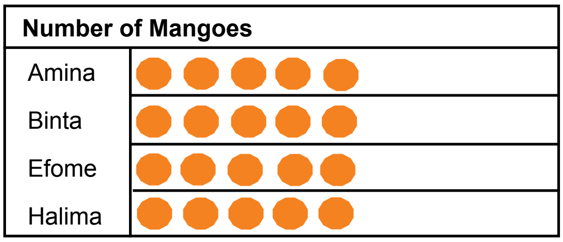
Section 4:
Mean Average

Amira and her sisters went searching for ripe mangoes. Amira and Halima each picked 4, Binta found 8 and Efome only 2. The pictograph on the right shows the number of mangoes they collected.



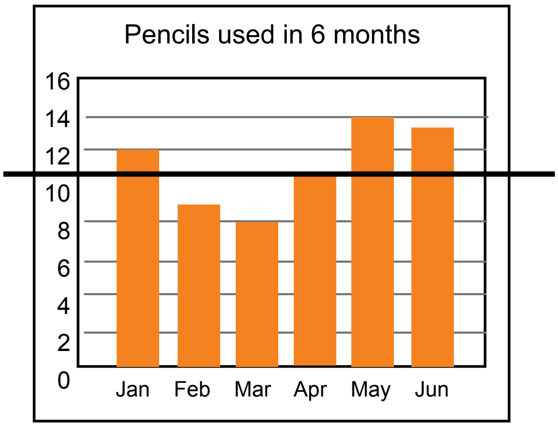
The sisters decided to share the fruit equally, so Binta gave Amira and Halima one each from her mangoes and she gave three to Efome.

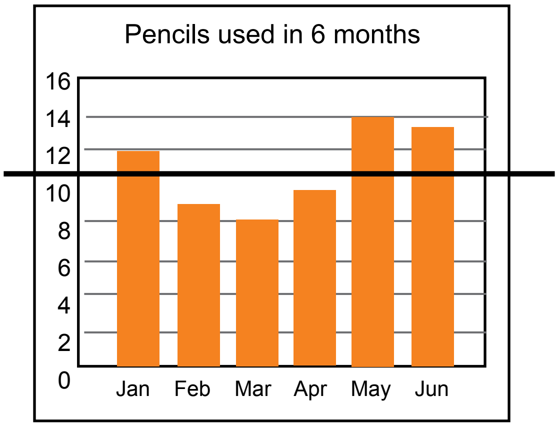




These two pictures illustrate the sharing so that everyone had the same amount. You will notice that the girls had collected 20 mangoes altogether. When these were shared equally, the four girls had 5 mangoes each.

Look again at the bar graph from Section 3 showing the number of pencils used in six months. It is copied twice below. How many pencils do you estimate will be used in in July? The black line on the graph on the left suggests that about 10 pencils, on average, are used each month: in some months more were used; in some months less were used. The line on the graph on the right suggests that about 11 pencils are used each month. Which line, do you think, gives the better idea of the average number for the six months? How can you decide?





The graph on the left shows February with one pencil below 10 and March with two below 10: a total gap of three pencils below the line. There are a total of ten pencils above the line for the other months. The graph on the right shows a bigger total gap (five pencils) for the months below the line and a smaller total (six pencils) for the months above. So the graph on the right, with the average bar at 11, shows a more even distribution across the six months.

Another way to consider the average for the six months is to calculate that, altogether in the six months, 67 pencils were used. That would have been equivalent to an average of 11 pencils each month (and 1 extra pencil).

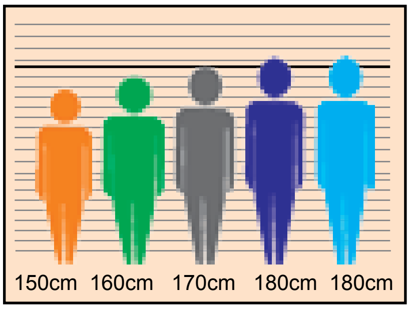
On average, we use about 11 pencils every month …

… so that's about 132 each year.

Some months the number used will be more than 11, some months will be less than 11, but 11 per month is a good guide to our usage.

The word ***average*** describes what is typical or normal for each month.

In a similar way, you can use an ***average*** **number** to describe what height is typical for an adult person. The graph on the right shows 5 adults. They have an average height (the black line) of about 170cm. Two of the people are more than 170cm tall. Two are less than 170cm.



When you add up all the heights of these 5 people, the total number of centimetres is 840. If that number could be shared equally by the 5 people, then each person would be 168cm tall. As the black line suggested, the average height of this group is about 170cm. The calculation shows that it is 168cm.

You may know that the average height for men in Nigeria (164cm) is a little taller than the average height for women in Nigeria (158cm). Different groups have different averages.

Mohammed's family are not average in height compared to other Nigerians. All the men and the women in his family are tall. Asibi is the shorter woman and she is 178cm tall. Her sister Rahama is 179cm. Saleh is 182cm tall and Mohammed himself is 185cm. They are all close to 180cm. When you add up all their heights,

the total number of centimetres is 724. If that number could be shared equally by the 4 brothers and sisters, then each person would be 181cm tall.

Some of the adults in Mohammed's family are less than 181cm tall. Some are taller than 181cm. But 181cm is the average height of the adults in their family. You wouldn't describe them as being of typical height because most adults in Nigeria are about 160cm. The average height for adults in Mohammed's family is taller than the average height for Nigerians in general.

Many people are shorter than 160cm and many people are taller than 160cm. But most adults are close to 160cm. *Are you taller or shorter than average? How close are you to the average?*

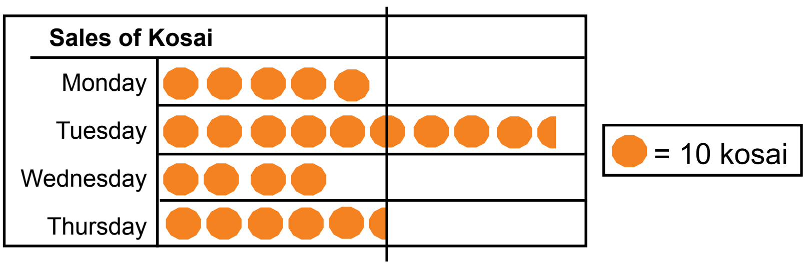
Mohammed's father Hassan keeps goats. On average, he has a herd of about 60 animals. Sometimes, when new kids are born, there are more than 60 goats. At other times when Hassan sells some goats or kills some for meat, his herd is less than 60. The size of his herd goes up and down but 60 is its average number.

When fully grown, the average height of Hassan's goats is about 55cm. Some fully grown goats are shorter than this, as small as 50cm; others are taller than this, as tall as 60cm when fully grown. They range from 50cm to 60cm but the majority are close to 55cm.

The notion of an average height is quite a sophisticated idea for Year 5 pupils. In contrast to the idea of sharing mangoes, you cannot, in practice, share heights. So, for year 5, you are advised to teach about the averages which represent physical objects being shared.

Here is one example about Amira's mother, Safiya, who you met in Section 2. She fries fresh kosai and sells them in the market. How many should she make for Friday?

The average sales are about 50 or 55 each day. Occasionally, she sells more than 55; some days she sells less than 50. So, for Friday, you would probably think that



she should prepare enough beans to make at least 55 or 60 cakes. The advantage for Safiya in this context of frying kosai at the market is that she can be flexible; she can fry less if she has fewer customers. But she needs to have enough beans prepared in case the demand is high.

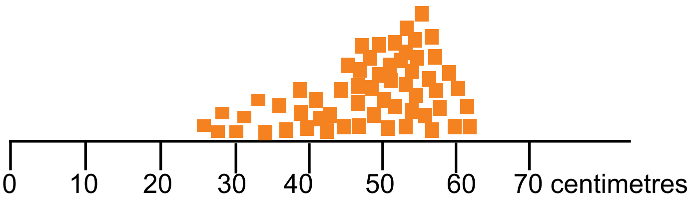
When you calculate how many kosai she sold in the four days Monday to Thursday, the total was 240. If she had sold this number equally on each of the 4 days, the average number was 60. Her average sales indicate that she should be prepared to sell 60 kosai on Friday.

You will learn in Section 5 that there are other indicators of a typical average amount of a group. This Section 4 is focusing on the idea of sharing quantities equally to discover the number that will represent a typical or normal amount for the context being looked at. This number is called the ***mean average***. You can find the mean average, as we have done with mangoes and pencils, by looking at a pictograph or bar graph and mentally adjusting the numbers. But the most efficient way of finding the mean average is to total all the figures and divide by the number of groups or figures that you have added.

For the families of Mohammed, Rahama and Asibi who you met on the previous pages, they have 9 children, 5 children and 1 child respectively. There are 15 children altogether in these 3 families. If that number were to be shared equally, each family would have 5 children. 5 is the mean average for the number of children in their families at the present time. Obviously, only Rahama has 5 children but the mean average suggests that this would be the typical number of children: some families will have more children; some will have less. In fact, if you did this calculation for all the families in Jigawa, Katsina and Zamfara, the mean average per household (in 2016) is 6 children.

In Year 6, pupils can begin to understand the theoretical aspects of a mean average. But this can still be challenging for pupils to comprehend. For example, if you calculate the mean average for the number of children in all of the families in all of Nigeria, the average is 4·5. But no-one can have half a child, so this theoretical idea of a mean average is difficult for many pupils to understand. For your Year 6 pupils, let's go back to Hassan's goats. Hassan measured the height of all his goats. For each goat, Mohammed wrote its height on a small piece of paper. They placed each height in position on a number line. Here, on the right, number line. On the next page is a picture of their result..

The heights of Hassan's goats range from 25cm to 61cm. Most of the heights are close to 55cm. So Hassan says their average height is 55cm. When Mohammed added all the 60 heights, the total number of centimetres was 2924. He divided this total by 60 and calculated a mean average to be 48.8cm. *Why is Mohammed's mean average lower than Hassan's estimate of 55cm ?*



Think

Were you able to explain why Mohammed's calculation of 48.8cm for the mean average of the height of the goats was much lower than Hassan's estimate of 55cm?

Could you explain to children how 60 is the mean average number of goats in Hassan's herd?

Watch

Watch the video clip MM9V4 on your phone. As you watch the lesson extracts, think about how the teacher explained the meaning of "mean".

Reflect

- How did the teacher manage to explain how to calculate the mean of 7, 13 and 10 without writing anything on the chalkboard?
- The children in this class clearly understand how to calculate the mean of three numbers when the totals of the three numbers were 30, 15, 21, and so on. How would you help them to find the mean of the four numbers 20, 23, 26 and 29 ?

  What difficulties might this calculation cause?
- Why might this class not have an understanding of "mean" as an **average**?

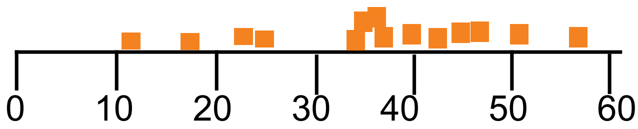
Work with your partner in school

Discuss with your partner how you can use an everyday context for pupils to find the mean average of a group of objects. Your lesson will need to provide

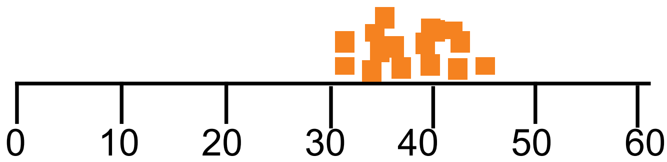
- an understanding of what an **average** represents;
- how to calculate the mean average by totalling all the objects and sharing them equally.

One such lesson involves the purchase of some wraps of groundnuts. (This suggestion will involve a small cost, unless you are lucky to have a pupil in your class who sells groundnuts after school.) You will need one wrap of groundnuts per group of four or five pupils. About 15 wraps would be ideal.

Draw two number lines from 0 to 60 across the chalkboard. Give each group of pupils one wrap of groundnuts and some small squares of paper. Without opening the wrap, the pupils must estimate how many groundnuts they think are in their small bag. They write their estimated number on a small paper square. Each group will then pin their number in the correct position on, or close to the first number line.



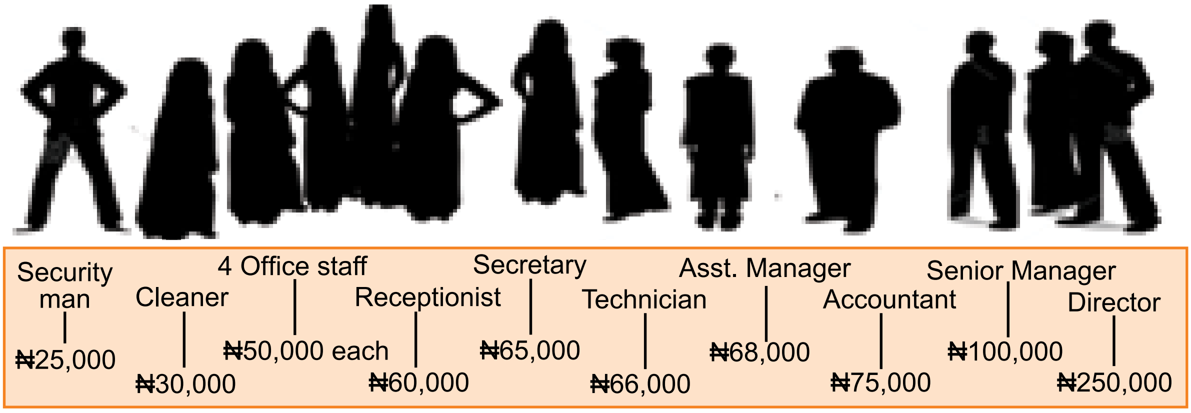
With all the estimates in place you can discuss what most people thought and the range of their estimates. In this example, fifteen estimates were made, ranging from 12 to 54 nuts. Most estimates were near 30-35. Then pupils open the wraps and count the nuts in their bag. Groups write the actual number on another small paper square. These actual numbers are now pinned to the second number line.



The actual numbers are not likely to be the same but they will be close to one-another. In this example, all fifteen wraps had close to 40 nuts. The range of nuts this time was much narrower, from 32 to 44. Discuss with the pupils what they now think is the average number of nuts in a single wrap. The final part of this lesson is to calculate the total number of nuts in all the wraps. Dividing this total by the number of wraps will give the mean average for the contents of a bag of groundnuts. In the above example the 15 wraps had a grand total of 555 nuts. So the mean average was 37.

Section 5:
Median, Mode and Range

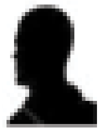
Meet all the people who work at Northern Enterprises and see what they are paid.



What do you think is the average pay?

When the workers reported that they did not get paid enough, the Director added all the salaries and divided by 13. He said the mean average wage is ₦72 000. The workers didn't agree with him.

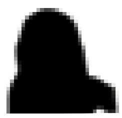
72,000 Naira is an excellent average salary.



How can the average be 72,000 Naira? You only pay two staff more than that. The other ten people all get less than that.

The biggest group of us gets only 50,000 Naira !

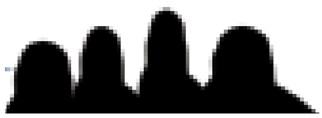
The receptionist said that her ₦60,000 salary was the average pay.



I'm in the middle. Six people get more than me. Six people get less than me. So my salary is the average.

But the office staff insisted that the average pay was only ₦50,000 because that was the most common salary. Four people had this amount, so they thought that ₦50,000 was the most typical salary amongst all the staff.

More people get 50,000 Naira than any other amount.



The story of the workers at Northern Enterprises exemplifies the difficulty mentioned at the end of Section 4. It is difficult to express what is actually meant by the term *"an average".*

Do you mean the average that would be obtained if all the money could be shared equally?

Do you mean the average that is the amount in the middle?

Or do you mean the amount that is most common and typical of what the majority get?

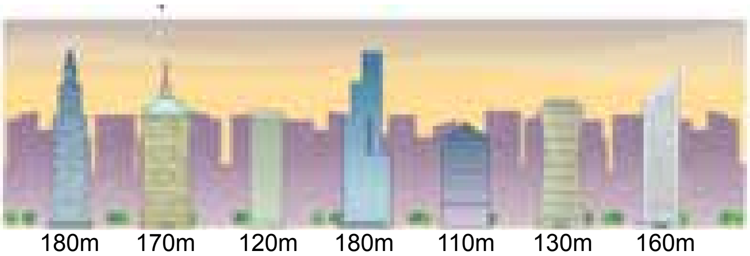
Because of this difficulty in defining one single average, mathematics has four different ways of defining the quantity which best represents an average amount.

You have already learnt how to calculate the first type of average average height for this group. But a new tower has just been built: – the mean.

To calculate the mean, you need to find the total of all the items and then divide by the number of items. This was the focus of Section 4.

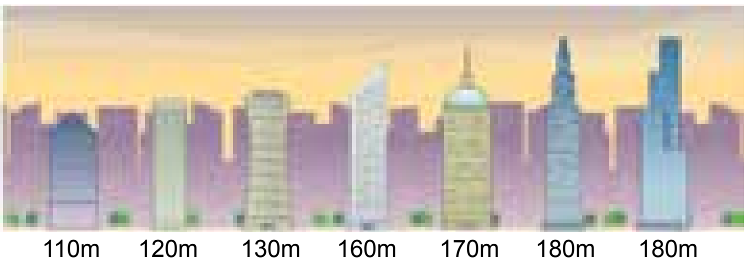
The Median

The second type of average is the one which describes the size in the middle – the median. To find the average called the median, you need to find the middle item of your data. In Northern Enterprises, the receptionist was the middle person and so the median value of the pay for staff at Northern Enterprise was ₦60,000.



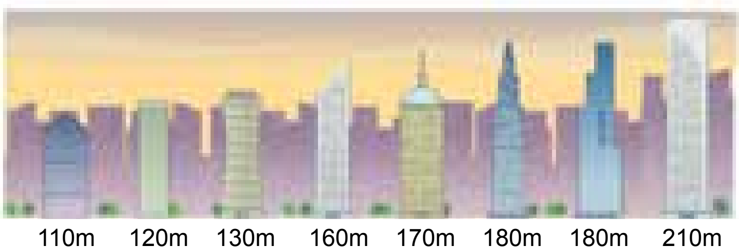
picture courtesy of BBC Bitesize

Seven new tall buildings have recently been built in this capital city. Which building is the median average height of these tall buildings? You could answer this question visually but it is much more efficient to arrange the buildings in order of size: the middle one is then much easier to recognise.



picture courtesy of BBC Bitesize

Now you can see that the grey trapezoidal building with the height of 160m is the middle one. So the median height is 160m. Well, in fact, you don't need the drawings to show you this median. You can find it by just writing all the heights in order of size and selecting the middle one. There are three new buildings taller than 160m; and there are 3 new buildings shorter than 160m; so the grey building is at the median height. It is the average height for this group.



When a new taller building is added, there is no longer a middle building in this group which can be the median height. Because there is an even number of heights in the list, the middle is now halfway between the two central buildings. So, in this case, the median height is 165m. The height 165m is the average of this group because there are four buildings taller than 165m and four buildings shorter than 165m. The median value is halfway between the two central measurements.

After Northern Enterprises bought more computers and decided to employ another technician at ₦66,000, they now have a staff of fourteen people. The median salary for the staff will now be the middle between the salaries of the seventh and eighth employees (when arranged in order). The seventh and eighth employees are the receptionist who earns ₦60,000 and the secretary who earns ₦65,000. So the median salary for staff at Northern Enterprises is now ₦62,500. Seven people get more than this; seven people receive less than this.

The Mode

The third type of average is the mode. This is the most common data in a sample – the most frequently occurring item or number.

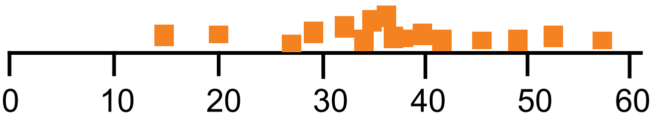
The modal average for the pay at Northern Enterprises was ₦50,000 because this is the most frequent amount of salary paid. Four people each receive this amount.

This graph shows the number of shoes sold by the general store in Taguna town. The store manager keeps a record of all the shoe sizes she sells. In this way she knows how many pairs of shoes to order because she knows which sizes are needed by her customers. You can see that she sells more of size 37 than any other size. She sold 20 of this size. 20 is the frequency, the number of pairs sold. It is size 37 which is the mode.

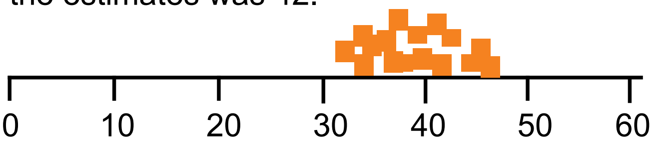
The mode is the simplest average to find because no calculation is needed. The mode is simply the most frequently occurring item or measurement.

The Range

Did you try the lesson suggested in the notes to Section 4 in which pupils are asked to estimate and then count the number of nuts in a wrap of roasted groundnuts? The results of one lesson were illustrated, using a number line. Here they are again.



The first illustration shows what the pupils thought that the number of nuts would be. The nuts had not been counted and there was a wide range of estimates from 12 to 54. Subtracting the lowest estimate from the highest estimate, you can see that the range of the estimates was 42.



When the nuts in the several bags had been counted, the numbers were found to be much closer to the mean average of 37. This time the range was only from 32 to 44. A much narrower range of only 8.

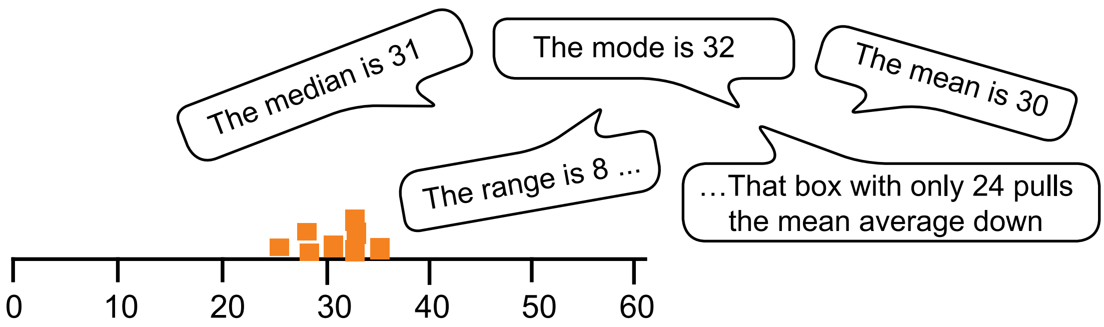
When the range was 42, the numbers in the sample were spread out. When the range was only 8, the numbers from the sample were bunched much closer together. Hence, the range of the data can also gives a sense of an average. It tells you how close the data is to a representative value for that data. The smaller the range, the closer together the data is to an average number. This means that a small range gives a good pointer to what the average amount is.

Choosing an average

Yusuf sells boxes of matches from his store in the village. Sometimes people complain that the matchsticks in the boxes are not enough. In case customers think he has been removing some of the matches, Yusuf decides to open some boxes to investigate the number of matchsticks in each box. The picture above shows how many he counted.

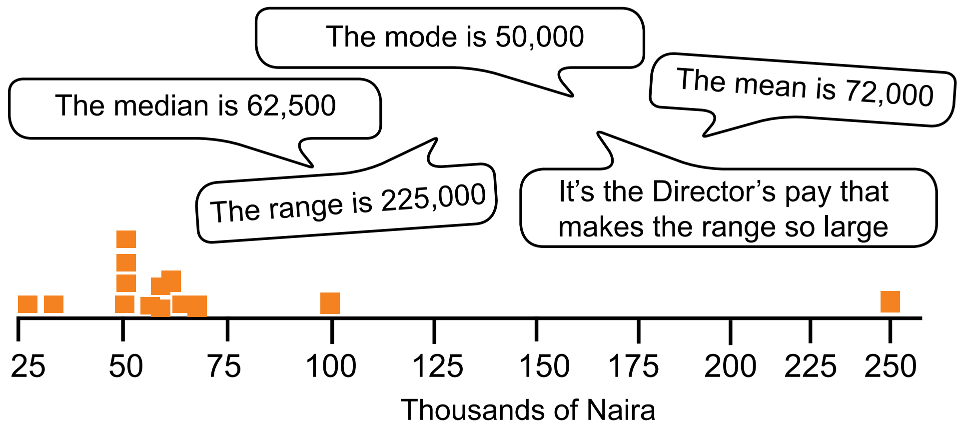
To study the results, Yusuf writes the numbers in order, smallest first.

24 28 28 29 31 32 32 32 34



Yusuf realises that sometimes his customers may get a box with fewer matches inside. The box with 24 matches is not close to any of the averages for the group. The other boxes are all close to the mean. It doesn't seem that the different types of average give very different information about the average number of matches in a matchbox. Each average gives similar information.

In contrast, the different types of average give very different information about the average pay at Northern Enterprises. What do you think causes such differences?



Which types of average, do you think, will Year 5 and Year 6 pupils find easy to calculate?

Which types of average will they find easy to understand?

Think

Match the types of average with the method for finding the average of a set of data.

- The Mode
- The Median
- The Mean
- The Range

- Arrange the numbers from lowest to highest and find the middle of that list
- Subtract the lowest value from the highest value in the list
- Find the most frequently occurring item
- Add all the numbers and divide by the number of items

Watch

Watch the video clip MM9V5 on your phone. As you watch the lesson extracts, think about the context which the teacher used to introduce the work.

Reflect

- What do you think were the objectives of the lesson? Were these achieved?
- The mode and the median which the children found from the teacher's data and from their group exercises were always the same pair of numbers. Is this an advantage or a disadvantage in helping pupils to understand the roles that the Mode and the Median play in Statistics?

Work with your partner in school

Discuss with your partner

- why re-arranging the numbers is important for calculating the median;
- which real data about the children in your class could give the pupils an understanding of what the Mode, Median and Range are measuring;
- how familiar objects, such as yams in the market; … a recognisable group such as a football team; can all be used in a lesson to discuss the Mode, Median, Mean and Range of data.

Summary of Module 9

In this module we have discussed the aspects of Mathematics which are essential tools of handling data – from the statistical question that needs to be answered to the interpretation of its answers. In Section 1 you learnt about recording tally marks and the use of a tally table for recording information as it happens. You know that tallies are grouped in fives to enable quick totalling of the data recorded. The focus of Section 2 was the use of pictograms and how symbols are used to display large quantities of data as well as smaller numbers. Hence, you understood that the pictograph needs a key to explain what the pictogram represents. In Section 3, we discussed bar graphs. You saw that bar graphs are a visual presentation of data to make it easy to understand. They range from simple block graphs in Year 4 to more sophisticated bar graphs in year 6. Whatever its level, data needs five things for it to be easily understood in a graph: an appropriate **title**, **categories** with **labels** and a sensible frequency **axis** to enable the **bars** to be "read". The scale of the axis is chosen according to the numbers which the graph is going to display.

In Sections 4 and 5 we discussed and interpreted data to understand the different ways in which mathematicians can describe the typical features of a group. You understood how to calculate the mean by sharing all the numbers equally to get a sense of an average quantity. You saw that the range of values contributes to a sense of the average by telling you how spread out or how central the data is. You also learnt that there are other ways of describing what is typically average for a group: the mode is the most frequently occurring item and the median is the central value.

When you looked at the graphs of Populations, Rainfall and the Food of Owls, you realised that in lessons on statistics, children not only learn about the mathematical skills but the data can also help them learn more about the world in which they live.

Ideas to try in the classroom

The data handling cycle attempts to provide the mathematical skills to answer a question which requires the analysis of information. So it is a good idea to link all five different sections of this module into one exercise which can attempt to answer a simple question. This may take three or four lessons of a week to carry out and will require the pupils to take some initiative.

There are many questions which can be asked about the children's own environment. You can suggest one for them; or the pupils may like to suggest their own. Here we will suggest that the class investigates the average length of a banana.

No need to buy a lot of bananas: ask children to measure any banana that they eat or see so that they can gather the data over a period - a week's homework could be for everyone to find and record the length of 10 bananas.

First, get a rough idea of how big the range will be: *perhaps 10cm to 20cm*? Pupils will make a tally chart listing all the numbers from 10cm to 20cm. (You will need to agree on whether pupils measure straight from end to end or measure round the curve – everyone must measure in the same way.) At the end of the week's investigation, collect all the pupils' results together so that you will have information for about ten categories. For Year 4 or Year 5 pupils, collect the measurements to the nearest centimetre and agree on the number of bananas they have found measuring 10cm, 11cm,12cm, and so on. For Year 6 pupils, the information can be collected as grouped data: 10 -10.9cm, 11 -11.9cm, 12 – 12.9cm, and so on. Next, the collected information is arranged into a bar graph. Choose the frequency axis according to how many bananas are in the largest category.

Finally, pupils will analyse their data with the help of their bar graph. They will be able to report on the mode, the median, the mean and the range of lengths which they have found.

They will be able to write

"In our survey of □ bananas we have found that the average length of a banana is □ cm."

- Collect some other questions which you think pupils will be intrigued to find an answer for.
- Collect some graphs which provide interesting information. Add them to your bank of teaching tools.
- Collect some interesting pictures of groups of people to provide a stimulus for investigating an average. For example, your class: what is their average age? … their average height? … etc
- Collect some statistics of your favourite sport. What is the average age of your favourite football team? How many goals does a player score, on average, in a year?

Experiencing change in the classroom

It is beneficial for you to note in your journal your experiences during teaching. It is also a good habit for teachers to observe and note if pupils are learning what you wanted them to learn. The following questions will be of help to you while writing of your experiences in the journal:

- Which activities did you try out in the classroom?
- Which activities went well? How do you know? Why did they go well?
- Which ideas were less successful? How do you know? Why were they less successful?
- If you try these activities again, what changes will you make to ensure that pupils' learning improves?

Suggestions for the next Cluster Meeting

Use the space below to write any notes about what you would like to discuss in the next cluster meeting. Your suggestions can include comments on challenges or simply an experience that you would like to share.

Your fellow teachers will like to hear about any statistical questions that your pupils answered. So be prepared to tell your colleagues about how you organised the children to collect any data they used.

Module 10: Measurement

Module 10:
Measurement

Numeration enables us to answer the question *"How many?"*

Measurement enables us to answer the question *"How big?"*

To know the size of things, we measure **length/ distance / height / depth** how long? how far? how tall? how deep? how wide? **weight** how heavy? **capacity** how much space? **time** how long will it take?

Each day there are many examples when you need to know how big something is … when walking, when cooking, when buying, and so on. Many of the measurements that we make every day are estimates …

How long will it take to cook the yam?

How long will it take to walk to the market and back?

Will that piece of wood be long enough to repair the fence?

How much water do I need?

… but other measurements need to be accurate … how much medicine to take? … how long and how wide is the piece of glass needed for the window?

In Module 8 you thought about Estimation. In this module you will be focusing on the teaching of accurate measurements. Pupils in P1-3 were introduced to the use of non-standard measurements for estimating the height of a table, or the width of the classroom, and so on. They used short units like their *hand-span* to measure a chair. They used longer units like a *stride* to measure the width of the playground. They found how many bottles of water are needed to fill a bucket. Doing such practical exercises provided a very important conceptual step for young children. But P4 pupils will understand that these non-standard measures cannot be reliably accurate. Measures like pace, hand-span, arm and foot vary according to who is making the measurement. A tall pupil will say that the classroom block is 32 paces long; a short pupil will say that it is 37 paces long. This module is concerned with ensuring that pupils know the standard units which are used for measuring.

The first steps in each section will help pupils to understand that one basic unit, on its own, cannot be suitable for measuring all sizes. For example, while you will teach them to use metres (for length), grams (for weight), litres (for capacity) and hours (for time), a metre is much too small to measure the distance between two towns and much too large to measure the thickness of a piece of paper. So each section in this module will refer to the range of units to use for large and for small sizes.

It will be important for Year 6 teachers to enable pupils to recognise and to understand the common features which the international metric system uses (the prefixes *milli-, centi-* and *kilo-*) to provide a range of small and large units for each measure. We will refer to this again in the summary section of this module.

Pupils will find it relatively easy to measure lengths and to have a good sense of the size of *centimetres* and *metres*. However, it is much more difficult for pupils to have an understanding of the other measures. This is because their bodies allow them to have an intuitive idea of distances: for example, they can imagine how many paces are needed to cross a road because they can see the length of their stride and the length of a metre rule. But pupils cannot see a weight, or time, and it will require the experience of volume over several years to recognise a container's capacity in litres. You will think about how to deal with this problem of a sense of size in the appropriate sections.

By the end of this module, you will be able to guide pupils to :

- measure lengths using standard tools of measurement and a range of units (Section 1)
- measure weight in grams and kilograms (Section 2);
- calculate the area of rectangles using square units (section 3);
- find the capacity of containers and measure volumes in litres and millilitres (Section 4);
- tell the time on both analogue and digital clocks and calculate lengths of time (Section 5).

The sections of this module are each relevant for all teachers of Years 4, 5 and 6 because of the range of measures introduced, because of the development of the mathematics within each section and because measurement using the metric system is closely linked to pupils' developing understanding of working with decimals.

Section 1:
Length

While many languages have just one word to describe how far it is from one point to another, English has many: length; height; width; distance; breadth, depth; … Pupils in Year 4 will learn to use these words from the contexts which you give them. Whichever word is used, the length of an item is always the distance from one point to another.

The standard unit for measuring length is the **metre**. Its abbreviation is “**m**”. This fixed length is about the same as the stride of an average-sized adult or the distance from finger-tip to finger-tip of a P4 pupil’s outstretched arms. Pupils need to see this length – either a metre rule or a metre marked on the classroom wall. You can cut a one metre long stick if you don’t have a ruler – add it to your collection of tools for teaching mathematics.

Your first lesson is likely to be one in which pupils record a variety of lengths and distances using your metre stick: the length of the classroom; the width of the classroom; the length of the chalkboard; the length and width of the classroom block; the distance to the gate; …

1 metre

Having measured some lengths to the nearest metre, half–metre or, even, quarter metre, pupils will now have an idea of how long a metre is. How far up their body does a metre reach? Ask them to use their sense of the length of a metre to estimate other measurements:

the height of the classroom; the width of the gate; the height of a tree; the length of their house; …

You can set pupils tasks such as “estimate the distance from your classroom to the Head Teacher’s office. Now measure and check how close was your estimate.” Children can set one-another similar challenges but you will need a metre stick for each group.

To measure shorter lengths, the metre is divided into 100 centimetres. The abbreviation for **centimetre** is “**cm**”.

1m = 100cm

1cm = $^{1}/_{100}$ m

1cm = 0.01m

Common rulers are 30cm long so they are an essential tool for measuring short lengths up to 0.3m

For measuring very small lengths, or for measuring more accurately, each cm is divided into 10 millimetres. The abbreviation for **millimetre** is **“mm”**.

1 cm = 10mm

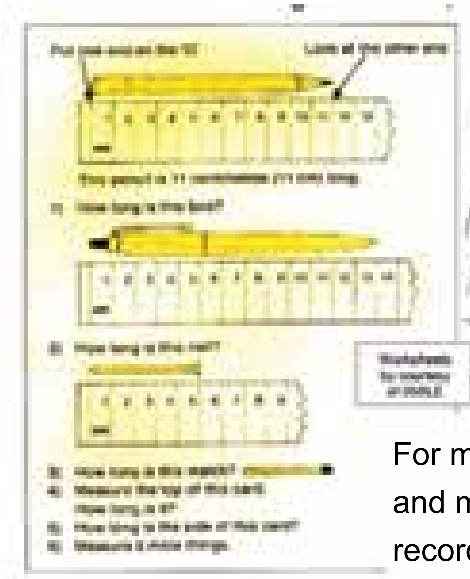
1mm = $^{1}/_{10}$ cm

1mm = 0.1cm

How many millimetres will be the same length as 1 metre?

Can you write 1 millimetre as a decimal value of 1 metre ?

When pupils use a ruler to measure short lengths, help them to know where to start measuring – some rulers have a zero right at the end, some have a short space before the measurements begin.



For more accurate measurements pupils will use cm and mm. Encourage them to use the decimal system to record lengths. A good activity is to ask pupils to draw a line; their partner then draws a line twice as long.
The two pupils check one-another’s measurements.

Pupils will enjoy recording the lengths of their bodies; there are many common ratios to find. From their chin to the top of their head; is it true that each person’s height is seven times the length of their head? From their elbow to the wrist; is this the same as the length of their foot? Pupils can use a ruler for this activity but a tape measure is much easier for them to use. Tape measures have the advantage of being flexible so they can be used to measure lengths like the waist or neck or hand size. They also have millimetres, centimetres and a metre all on the same tool. This helps them to have a sense of size of these basic measures of length.

Pupils will be unable to measure long distances but need to understand that using a metre stick or tape to measure the length of road from one village to the next is impracticable. Such large distances are measured using thousands of metres – kilometres. The abbreviation for **kilometre** is **“km”**.

1km = 1000m

1m = 0.001km

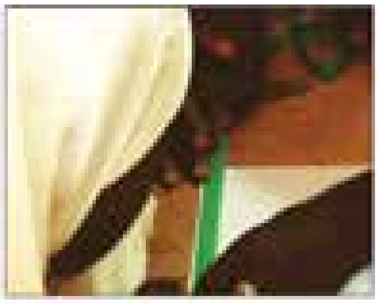
So a 100m race is $^{1}/_{10}$ km.

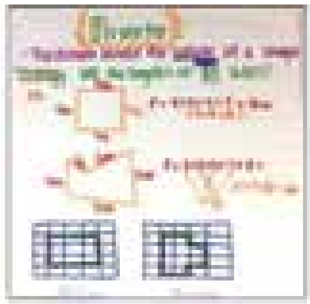
Pupils can visualise a 100m race track and so this may help them to have a rough idea of the length of 1km as ten 100m race tracks, end to end. Perhaps you can identify a distance for them that is 1km – from the school to the hospital or from the school to the market, …

Our table is 62cm wide

Our chalk-board is 3m long

Year 4 pupils will do lots of measuring and recording the lengths they measure. They start by writing the lengths to the nearest centimetre or nearest metre …





… then, later, they use centimetres and millimetres to measure more accurately. Help pupils to progress from writing 29cm 6mm to writing 29.6cm. Help them to progress from writing 1m 26cm to writing this as 1.26m.

In Year 5 pupils will progress to adding and subtracting lengths.

If it is 2.5km from a to b and 3.2Km from b to c, how far is it to walk from a to c, via b?

The top needs 1.2m of material and the bottom needs 1.9m, what length of material should you buy ?

If 4cm of a 17.5cm pencil has been used, how long is the pencil now?

Measuring the length and width of a rectangle will allow pupils to calculate the rectangle's perimeter. Having measured the perimeters of five or six different rectangles, ask pupils to generalise the process of finding the perimeter of a rectangle. Can they describe a short-cut to find the perimeter? Can they create a rule for finding the perimeter of a rectangle?

Measure all four sides and add them together

You can add two lengths and two widths

You can add the length and width and then just double

The formula $P = 2\ (L + b)$ then grows out of the pupils' experience of measuring and so the algebraic representation has meaning for them.

If the perimeter of the football pitch is 300m and its length is 90m, what is its width?

Year 6 pupils will do problem solving like this involving lengths. In particular, they will need to recognise that to add (or subtract) two lengths, the lengths must be measured in the same units. For example, to calculate how much wood strip is needed to frame a chalk-board 3m long and 60cm tall, you need to change the length to 300cm or change the height to 0.6m so that an answer of 7.2m or 720cm can be calculated for the perimeter of the rectangular board. Therefore, Year 6 pupils need to be able to convert metres to centimetres and centimetres to millimetres as appropriate to the context of the problem being solved.

Think

Thinking of your current classroom practice:

- have you used actual objects like those mentioned to teach pupils how to measure?
- has your teaching included real experience of pupils using mm, cm and m?
- do your pupils have any sense of how long is 1 cm, 1m, 100m , or how long is 1km?
- have you encourage individuals to make measurements?

Watch

Watch the video clip MM10V1 on your phone. As you watch, think about the following questions:

- how did the teacher explain the meaning of length?
- what measuring tools do the pupils use?
- how did the teacher encourage pupil participation?

Reflect

Following are two different lesson outline plans.
Compare the two plans and answer the questions below.

Lesson A

The teacher makes one central table and places measuring tools (30 cm rulers, tape measures, metre rule) with several different objects to measure.

- The teacher asks each group of pupils to come to the table in turn to measure the length, width, and height of each object.
- Pupils return to their groups and record their measurements of each object in a chart.
- Then each pupil writes five statements comparing the objects using terms like *longer than, shorter than, width, height, length* specifying by how many centimetres or millimetres the lengths are different.

For example
"The scissors are 2cm shorter than the pencil."

Lesson B

The teacher pairs up the pupils and gives each pair a measuring tape. The teacher writes the following on the chalk-board:

Height of pupil = ...
Length of head
(chin to top) = ...
Length of arm
(elbow to wrist) = ...
Length of foot = ...
Length of hand = ...
Width of hand = ...
Width of thumb =.......................

- Each pupil measures their partner and they both record the results.
- They share their results with their group and investigate any similar ratios that they discover.

1. Which lesson do you think will be more interesting and engaging for pupils?
2. Which lesson might have more challenges for you as the teacher? How might these be overcome?
3. Which of the two lessons would you like to try with your class? Say why.
4. Would there be an advantage to try both lessons with the same class?

Work with your partner in school

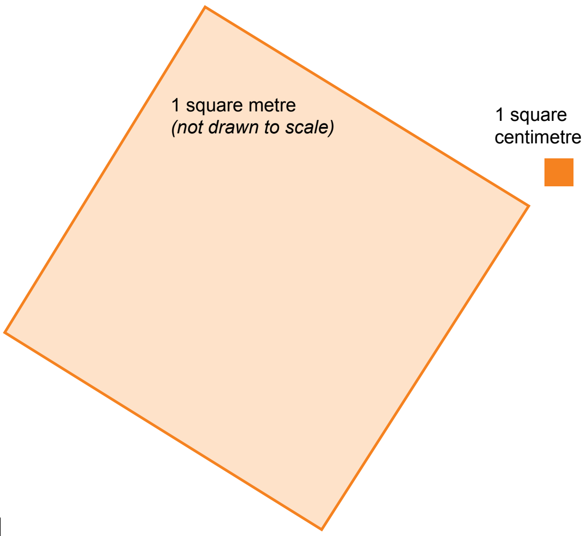
List ten items which you could make available in the classroom for pupils to measure. *teaspoon fork pencil seat door table book shoe chalk-board window eraser paper .*

First, you and your partner each make a guess of each item's length. *Did you use mm? cm? m?* Then you measure each item accurately. *Again, did you use mm?*

cm? m? Whoever makes the most number of closest guesses will be the winner of this competition.

- *Could you make this game an interesting classroom lesson?*

Section 2: Area



When pupils are familiar with measuring lengths using metres and centimetres, they will be able to draw a 1 centimetre x 1 centimetre square and a 1 metre x 1 metre square. You will know from the Module 8 section 5 where we discussed *Indices* that these squares are described by the abbreviations $1m^2$ and $1cm^2$ and, sometimes, as 1 square metre and 1 square centimetre.

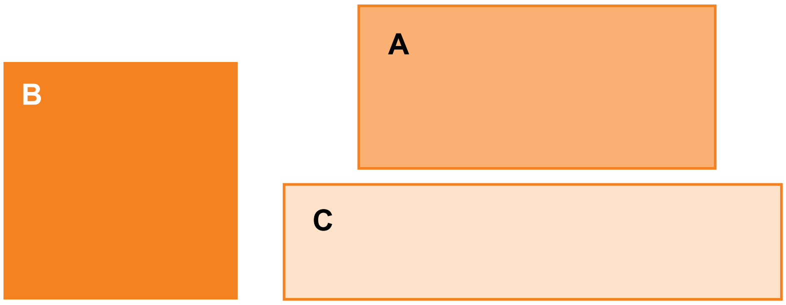
Making these squares from paper or cardboard will enable pupils to realise that the number of squares which cover a shape can be used to describe how large that shape is.

Pupils can use their metre squares to describe the size of their classroom. They can check which rooms are bigger than other rooms by finding how many square metres are needed to cover their floor area. Measuring the area of a shape is to measure how much surface it has.

An average size football pitch is 100m x 50m. That's an area of 5000 m^2 because it would need 50 rows of 100 metre squares to cover the pitch.

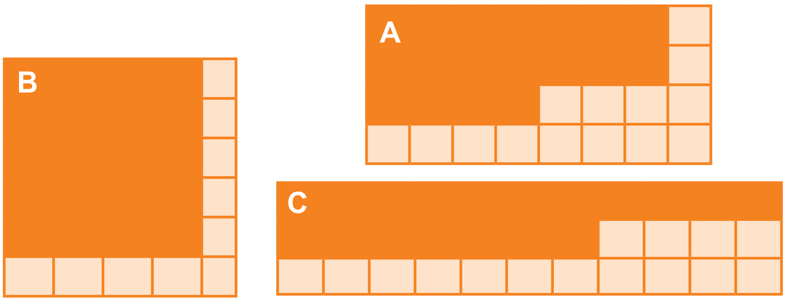
The area of a very large surface is measured using kilometre squares. Imagine a very large square sheet 1km x 1km. You would need 39,762 sheets like this to cover the whole of Zamfara. You could cover the whole of Katsina with 24,192 kilometre square sheets. For Jigawa 23,154 kilometre sheets would be needed to cover the whole state. So you can see that by knowing how many square kilometres an area covers, you can know which area is largest.

Here are three rectangles.
Which do you think has the largest area?



Their surface areas are similar. To know which rectangle has the largest area you will need to know how many centimetre squares can cover their surfaces.

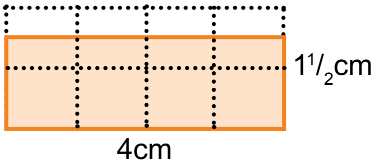
With just some 1cm squares drawn in, you don't need to draw all the squares.



There is enough information here to know that the rectangles A, B and C contain 8 rows of 4 squares, 6 rows of 5 squares and 3 rows of 11 squares – giving area measurements of $32cm^2$, 30 cm^2 and 33 cm^2 respectively.

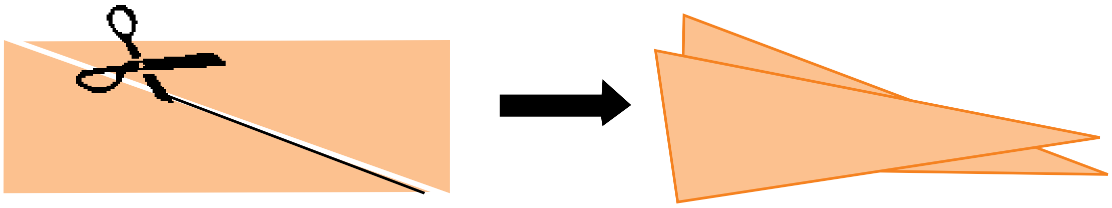
Through several such exercises finding areas of a variety of rectangles, pupils will learn that knowing the measurements of the length and the width of a rectangle is sufficient for them to be able to know its area. Let them tell you a suitable rule for finding the area of a rectangle.

By encouraging the pupils to tell you a rule for calculating the area of a rectangle (*instead of you telling them*) the pupils will be describing something that they have already discovered and understood. When pupils tell you that you don't need to count all the squares, they only need to know how many centimetres fit along each row (the length) and how many rows there are (the width), challenge them to tell you the area of a rectangle which has a length of 4cm and a width of 1½cm. Does their rule still work?



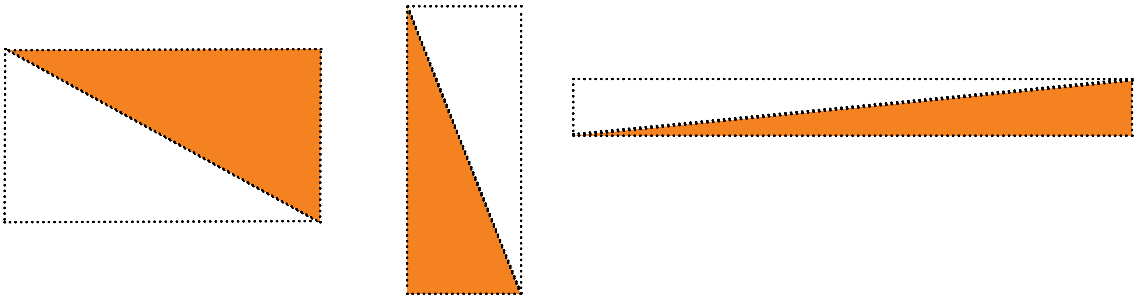
When pupils tell you that the area of the rectangle can be found by multiplying its length by its width, the formula which you agree, such as A = (l x w), indicates their ownership of this mathematical idea.

In Year 5 this ownership can be extended to find the area of any right-angled triangle. Pupils will recognise that every right-angled triangle is half a rectangle. They should discover this fact by cutting some rectangles in half along one of the diagonals, placing one piece on top of the other.



The placing of one piece on top of the other is an important step for Year 5 pupils because it helps to develop the idea of mathematical proof, confirming that the two parts are equal halves and justifying, conclusively, the visual impression suggested by the drawing. This simple act also supports the understanding that a rectangle has two pairs of equal sides.

The next step is for pupils to understand that a rectangle can be constructed around every right-angled triangle.

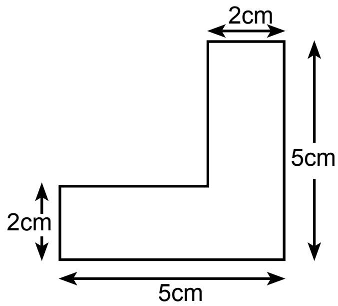


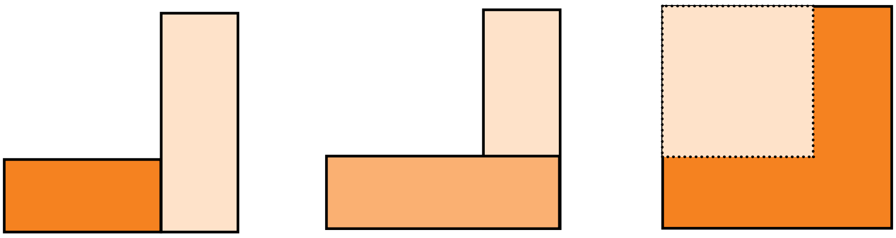
This fact can enable them to calculate the area of any right-angled triangle. They will draw several right-angled triangles and their enclosing rectangles. Ask them to find the area of the triangles. *What two lengths do pupils need to know to calculate the area of the triangle?* Once again, ask the pupils to tell you the rule in their own words and from their own experience. You may want to guide them to express their rule concisely but they will not find it difficult to say "multiply the length by the width and halve the answer". You may, however, need to specify which two of the three sides of the triangle are the two sides which correspond to the length and width of the rectangle.

Year 6 pupils will calculate the areas of rectangles and triangles, checking that their units are matching. Their area problems will include compound shapes such as this "L"-shaped area on the right.

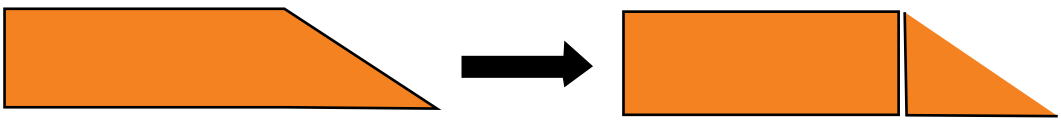
Note that you will expect Year 6 pupils to devise their own way of calculating the total area of such shapes.

To encourage the development of this ability, you will need to ask pupils to explore different ways of cutting the same compound shape into rectangles, including subtracting one rectangle from another. There are three possibilities for the “L”-shape.





The ability to cut a shape into rectangles or triangles (whose areas can be calculated separately) will allow pupils to find the area of a trapezium such as the following.



Think

1. What measurements would you need to know to be able to find the area of this trapezium?

2. If you use the formula Area = length x width for a rectangle 4½cm x 2½cm, the multiplication gives an answer of 11¼. Can you draw a diagram to explain where the ¼ comes from?

Watch

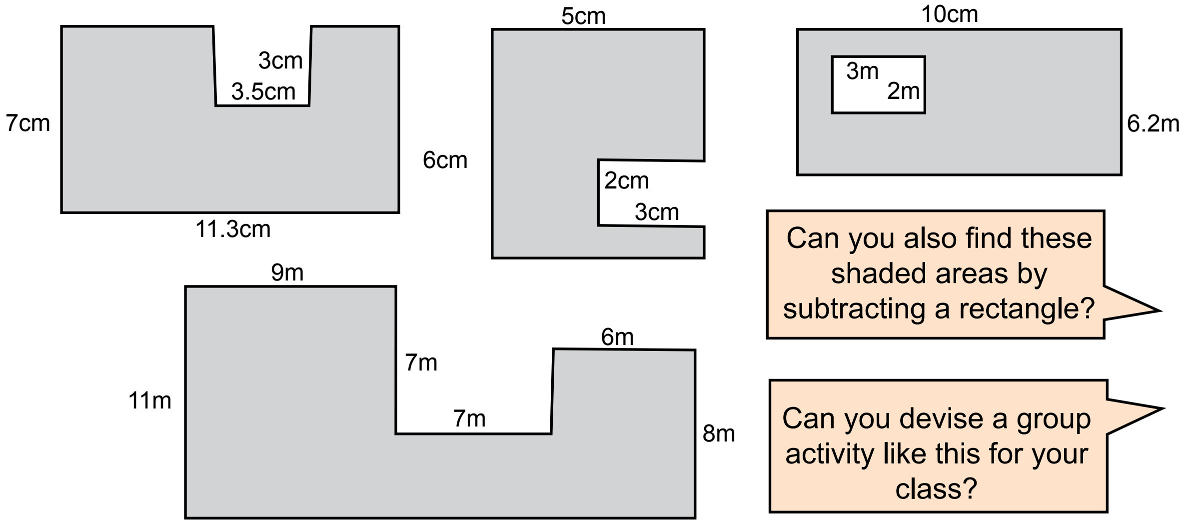
Watch the video clip MM10V3 on your phone. As you watch, think about why the pupils seemed eager to participate in this lesson.

Reflect

Do you think that the pupils in the video clip recognised that 18cm2 describes 18 small squares? *Explain your answer.*

Work with your partner in school

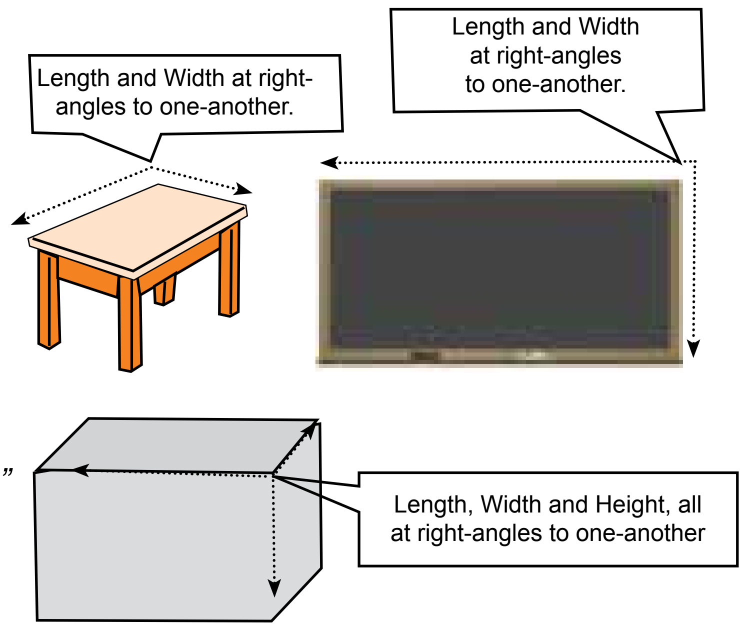
For each of the following shapes, you and your partner must find a different way of splitting the shape so that you can use rectangles to find the shaded area.



Section 3: Volume and Capacity

Volume

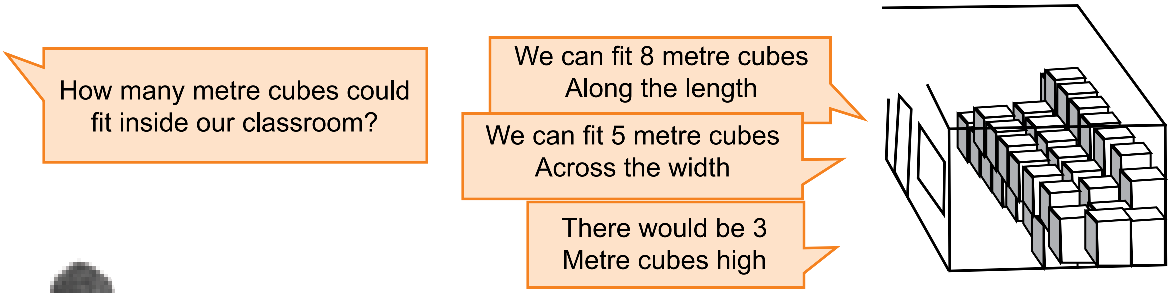
The practice of measuring *length in **two** perpendicular dimensions* has given us the units of square centimetre, square metre and square kilometre to measure surface areas. Measuring how many of these square units can fit an area gives an answer to the question *"How much flat surface does the object cover?"*

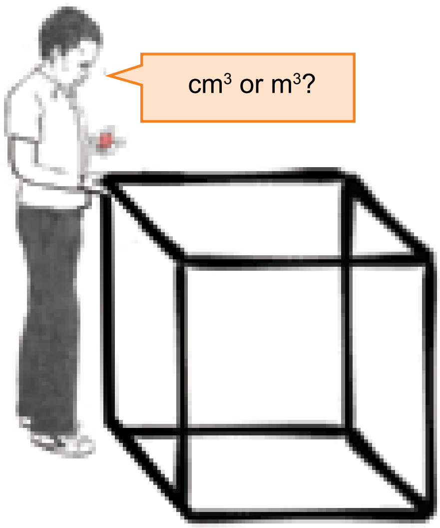


We now measure *length in **three** perpendicular dimensions* to provide the units of cubic metre (m^3) and cubic centimetre (cc or cm^3) to measure volume.

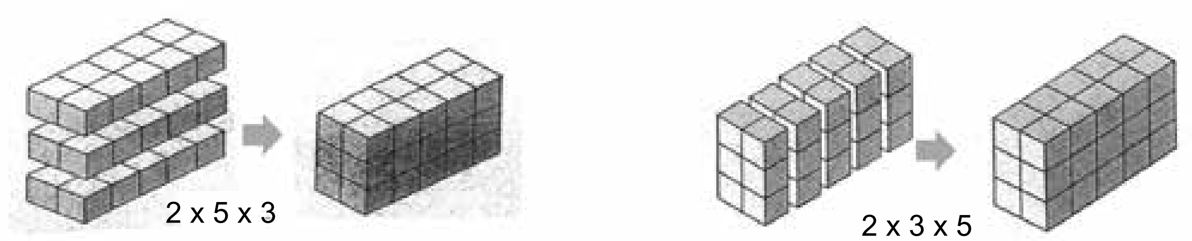
Measuring how many cubes can fit inside a shape answers the questions *"How much space does an object contain?"* or *"How much space does the object occupy?"*

Perhaps you can find, or make, a cardboard box that is a metre cube to help pupils to have a sense of the unit's size. If finding a large cardboard box is difficult, you could make the frame of a metre cube using twelve sticks which are each 1m in length. Seeing a metre cube frame will help Year 4 pupils to answer the question of how many cubic metres could fit inside their classroom. This will provide a basis for the concept of measuring the length, width and height to find the volume of the classroom. Pupils can also measure the length, width and height of their rooms at home to compare the sizes of the rooms.





For a smaller space or object, finding (or calculating) how many centimetre cubes can fit inside it will give a measure of its volume. Whether you use a cubic metre or a cubic centimetre to measure the volume will depend upon which size of unit is most sensible for the size of space that you are measuring. When pupils are able to see that the volume of a cuboid can be measured by knowing how many small cubes fit inside it they will develop the understanding that multiplying the cuboid's length, width and height is a more efficient method than counting. Encourage pupils to develop this understanding by drawing some diagrams like the following which, by showing the layers of cubes, explain why the multiplication gives a measure of the cuboid's volume as $30cm^3$.



Rather than tell pupils the formula V = L x W x H, ask them to do two or three questions like these in their groups. Without any equipment, you will need to draw a few diagrams so that pupils can visualise the small centimetre cubes inside the cuboid. Tell each group to agree on an efficient way to know how many small cubic centimetres are inside. Ask pupils to describe how they calculated the volumes. *Can they suggest a quick way of finding how many cubes are in one layer? So how will they find the number of cubes in all the layers? Does their method always work? Can the class write their method as a rule for other people to follow?* Guiding them to write the formula from their own experience will give pupils both a mastery and an ownership of the mathematical skill.

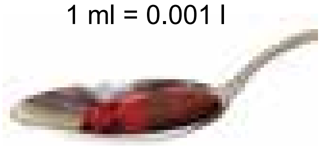
It will be an easy rule for pupils to discover because it is natural to describe solid objects like cuboids by measuring how long, how wide and how tall they are. This is not the case for liquids. As you know, liquids don't keep their shape like solids do. Liquids, and gases, take the shape of whatever container you put them in.

These containers all hold the same amount of liquid. In each container, the liquid has a different shape but its volume is the same. You can demonstrate this to pupils by pouring the same amount of water into three or four different shaped bottles or drinking glasses. We cannot consistently measure the length, width and height of a liquid unless it is in a rectangular cuboid container. For this reason, the volume of liquids (and gases) is measured in different units: **litres** (abbreviation l).

Pupils will likely be familiar with "litre" bottles of water and will readily understand this measure for liquid volumes. In common with the other units of the decimal metric system, measurements of smaller quantities are made using smaller units of measure. For small volumes, we use **centilitres** (abbreviation cl) and **millilitres** (abbreviation ml).

1 litre = 100 centilitres = 1000 millilitres

1 cl = 10 ml
1 ml = 0.001 l



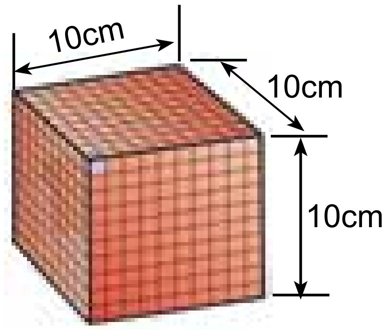
A millilitre of liquid is a very small unit of measurement. This teaspoon holds 3ml of medicine.

You may be able to bring a measuring jug to the classroom or use a science measuring cylinder to demonstrate to Year 4 pupils how volumes of liquid are measured using litres and millilitres. Pupils need to develop the skill of matching a container, such as a cup or glass, or a bucket, with a realistic estimation of its volume in millilitres or litres.

HOW MANY MILLILITRES OF LIQUID DO THESE CONTAINERS HOLD?

A litre is the same volume as a cube which is 10cm x 10cm x 10cm.

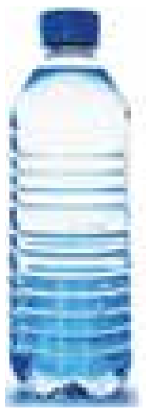
Knowing that 1 litre is the same volume as a 10cm x 10cm x 10cm cube, how many cubic centimetres have the same volume as 1 litre?



The result of this calculation will show you that 1 *ml*, being $^{1}/_{1000}$ litre or 0.001 litre , is the same volume as 1cc or 1cm^3 because the 10cm x 10cm x 10cm cube contains 1000cm^3. Cubic centimetres and millilitres are, therefore, alternative equivalent measures for small volumes. When you buy a can of drink, such as cola or lemonade, you may find the volume written on the can as 330cc, 330cm^3, 330ml or 30cl – these are all the same volume. However, you will find that litres and millilitres are the units mostly used to measure the volume of liquids (and gases) and cm3 are used for solids.

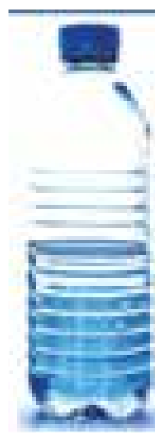
Capacity

You will also meet the word ***capacity*** when reading or thinking about **volume**. For young children the difference in meaning between the two words is quite subtle and so this distinction is probably best left until Year 5. "Capacity" refers to how much volume a container can hold. The measure of capacity, usually litres, describes the maximum volume that it can contain.



The bottle of water on the left has a capacity of 1.1 litres.
It contains 1 litre of water.
It is almost full.
There is a small gap with a volume of 100ml at the top.

The bottle of water on the right also has a capacity of 1.1 litres. Half of the water it contained has been used. The volume of water in the bottle is now only 500ml but the capacity of the bottle is still 1.1 litres. It could contain another 600ml.



Discovering capacity (and introducing the concept of measuring volumes) can be fun for Year 3 or Year 4 pupils as they discover how many bottles of water are needed to fill a container. In Year 5 measurement tasks are focused on using the standard measure of a litre to extend the notion of measuring liquid volumes to the understanding of how potential capacity can be measured.

Year 6 pupils will solve problems involving capacity and volumes so that they will be skilled at using decimal notation in the context of measuring volumes. For example, they will calculate how much more water can be added to a 2 litre bottle which already contains 1.35 litres.

Think

The word “capacity” is often used figuratively. Your classroom may have the capacity for seating 40 pupils and another room may have the capacity for 75 pupils. You may think that Hassan has the capacity to succeed and go to university. You may say “This girl has the capacity to do well.” *How similar is this use of the word to its meaning in mathematics?*

Watch

Watch the video clip MM10V4 on your phone. As you watch, think about the following questions:

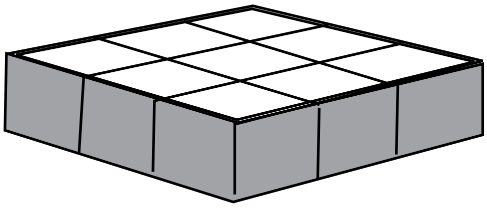
- Do the activities carried out by the teacher and pupils in the video clip make the learning about measuring volume and capacity more interesting and enjoyable? *How can you know?*
- What type of class organisation does the teacher use?

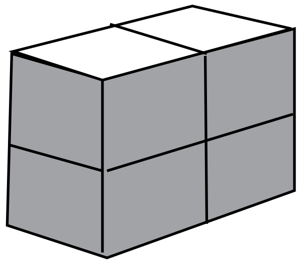
Reflect

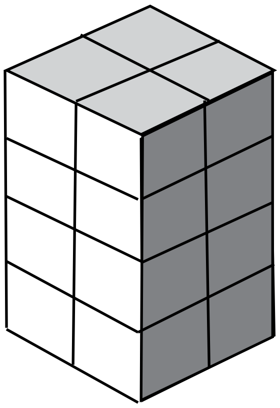
1. What do you think was the learning objective for this lesson? *Do you think that this learning objective was achieved?*
2. What had the teacher prepared to enable the success of this lesson? *Would the learning objective have been achieved without this preparation?*

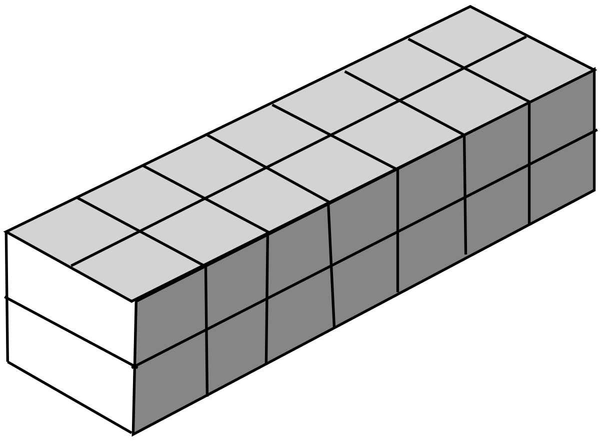
Work with your partner in school

1. It is not easy for teachers to draw supportive diagrams for this work in 3-dimensions. Spend some time with your partner practising drawing some cubes and cuboids so that you can guide pupils to visualise the small cubes inside larger cuboids. Try these:

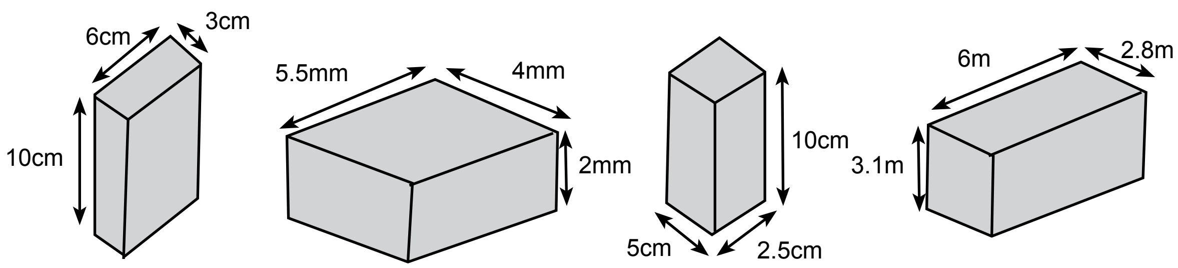








2. Now try drawing these outlines. Include the measurements so that pupils can calculate their volumes without seeing the individual unit cubes.



Section 4:
Weight

Pupils can see a metre length and have a mental image of its size. They can see a litre bottle of water and have a sense of the size of I litre of liquid. But they cannot have a sense of weight in a similar way - you cannot see *weight*. But you can experience it: carrying a bag of books; lifting a bucket of water; holding a bottle and knowing whether it is full of water or whether it is empty even without looking. However, a meaningful judgement of weight is difficult because the weight of an object depends upon what it is made of. Pupils who help their parents in the farm may know that a sack of sheep's wool is much lighter than a sack of yams. A sack of maize has a different weight than the same size sack of charcoal. A small knife can be heavier than a large piece of paper.

The standard unit of weight is the **gram**; its abbreviation is **g**.

A gram is much too small for children to handle. A teaspoon holds about 5 grams of sugar, so 1g is about the same weight as one fifth of a small sugar cube. Because it is a small unit, you may, instead, be more familiar with the weight of 1 **kilogram** – the weight of a large box of sugar cubes. The kilogram, abbreviation **kg**, is 1000 grams. This is quite heavy for young children to handle. Pupils may be more familiar with the weight of a plastic bag of cornflour, a cup of rice, a can of beans, a container of millet. You probably buy a butter tin of garri which weighs about 500g. Expect pupils to read *kg* as "*kilogram*" not "*kay-gee*". Similarly, pupils should read *g* as "*gram*".

The most easily obtainable guide for 1 kilogram is a plastic bottle containing 1 litre of water. Most commercial drinks, like Cola and Orange are available in cans of 330ml - sometimes in 500ml bottles. These are all useful for comparing weights because 1ml of water weighs 1g. A half-litre of water is 500ml and so weighs 500g. A can of Cola is a close approximation to 333ml or ⅓ kg.

You can help Year 4 pupils to gain a sense of weight by providing a range of objects, including a plastic bottle of water or a drink.
Ask each group in the class to guess which object is heaviest and guess which is lightest.

Then pupils hold each object and make a list of them in order of weight – heaviest first.

Check the groups' lists by using the bottle of water or the drink can as a guide to the weight of each item.

A gram is very small compared to a kilogram but a kilogram is also too small a unit to measure the weight of something very large such as a lorry or a shipping container. The load carried by a lorry, such as sand for building, is often measured by weighing the lorry when it is empty and then weighing it again when it has been loaded. This needs a special weighing machine that the lorry can drive onto. To measure such large weights, the unit we use is a **tonne**. This is not usually abbreviated. 1 tonne = 1000 kg. *How many grams are the same weight as 1 tonne?*

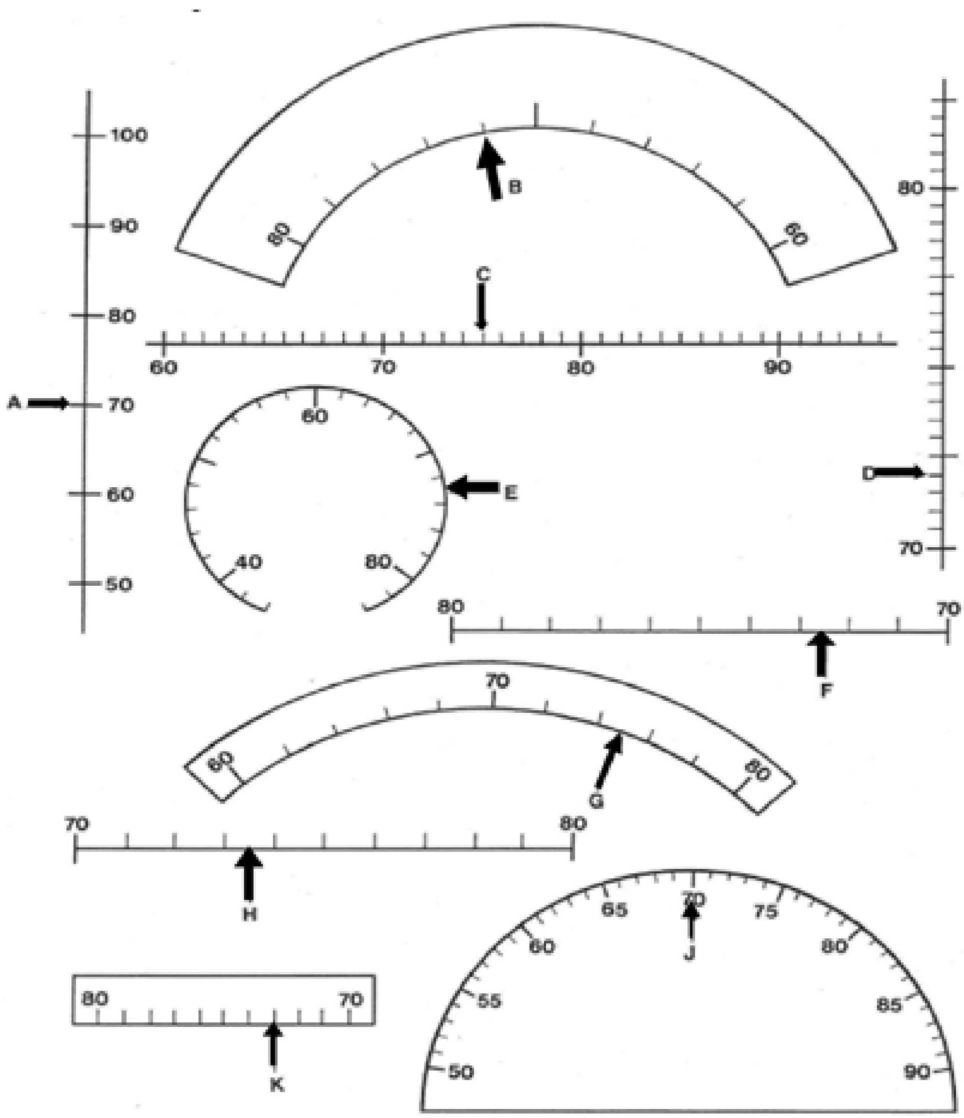
(pictures adapted from Scania advertising)

For Year 5 pupils to know a weight accurately, each object needs to be weighed using a scale.

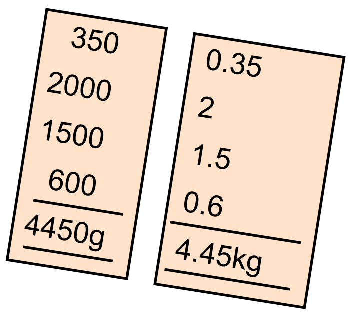
			
Balancing scales	Spring balances	Weighing macjine	Kitchen scales

There are different types of scales which can be used to measure weight. To be able to read the weight indicated by any of the weighing machines, pupils need to learn how to read a scale. This can only be done in the classroom if you are able to bring some examples. Otherwise, you would need to collect some pictures – perhaps you are able to copy some drawings like the following or use some illustrations from a text book. *Add some useful scale readings and weighing pictures to your collection of tools for teaching mathematics.*

The following picture (adapted from SMILE) shows ten scale readings. Four pairs of the readings show the same weight. *Which four?*



Year 6 pupils will learn how to solve problems and to calculate with the units for weight. As with the units for capacity and for length, they must be sure that their measurements are in the same units before adding or subtracting them. They will learn that this is often easier to do by using decimals.



- To find the total weight of Halima's shopping she needs to add

  350 g of garri

  2kg of sugar

  3 bottles of water each 500ml

  600g of millet

- If Mohamed's 2.35 tonne lorry weighs 5.1 tonnes at the weighing station, how much building sand is his load?

  If the sand costs ₦2 per kg to be loaded, how much should Mohamed pay for the sand?

Think

1. Thinking of your last lesson teaching about weight, which objects did you bring to the classroom to give pupils a real experience of estimating weights?

2. What is the most important skill required for reading the weight indicated by a pointer on a scale?

Watch

Watch the video clip MM10V2 on your phone. As you watch, think about the following questions:

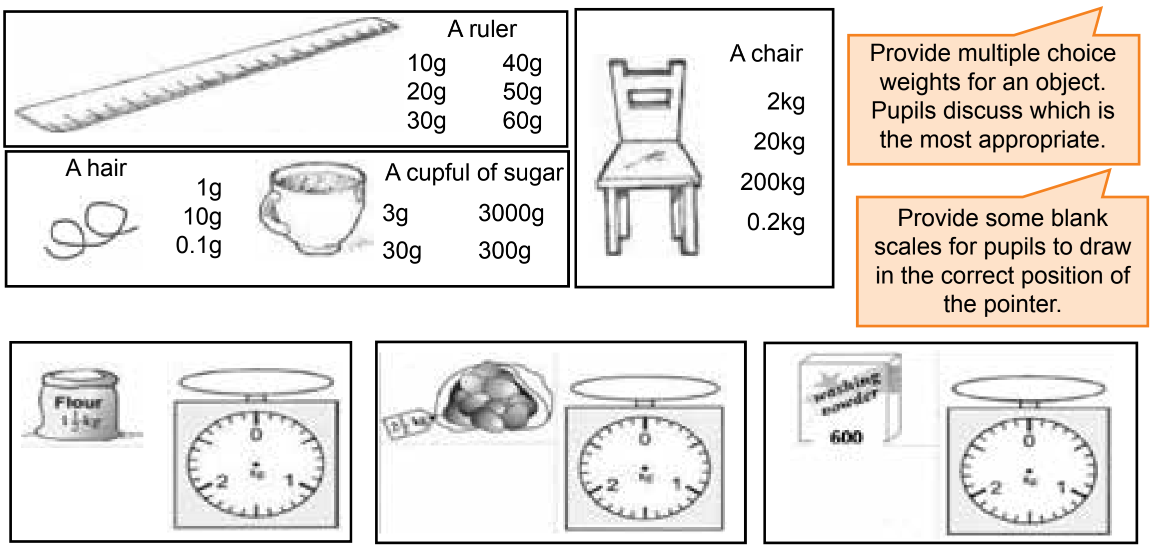
- did the teacher use any low cost materials in the lesson?
- did the teacher provide the children with any experience of measuring weight?

Reflect

If you were the Head Teacher observing the lesson shown in the video clip, what three issues about teaching weight would you want to discuss with this teacher?

Work with your partner in school

Because it is difficult to have sufficient equipment to teach about weight in a mathematics lesson, teachers often just demonstrate the theory about the gram and kilogram units. Discuss with your partner how you could use these two ideas to help pupils to develop a better sense of weight.



Section 5:
Time

Unlike the metric systems for measuring Length, Volume or Weight, measurements of Time are not based upon the base 10 decimal system. This is because our units of time are the result of the rotational movements of the Earth around the Sun (a year), the Moon around the Earth (a month) and the Earth around its own axis (a day). Our day is divided into 24 hours; each hour is divided into 60 minutes and each minute is divided into 60 seconds.

These different sized units of time provide measures for long, medium and short periods but they are not decimal factors or multiples of one-another. The different periods of time which pupils come to understand during P1-6 provide a good opportunity to link Mathematics and Science. Additionally, the defining of the lengths of days, months and years has an interesting history. During the teaching of mathematics, pupils in Primary School will learn about measuring Time via a progression throughout Years 1 to 6. Here is an overview of that progression:

Year 1 Order events of a day	Year 2 Hours and ½ hours Days of the Week	Year 3 Know time in hours/minutes The cycle of Seasons
Year 4 Read clock time, using am/pm Read calendar and write dates Time intervals	Year 5 24-hour clock times Digital clock times Solve time problems	Year 6 Seconds/minutes/stop-clock times Read timetables Converting between units of time.

This section on measuring time deals with the two aspects of time appropriate for Years 4 – 6: **Telling the time** and **Calculating a period of time**.

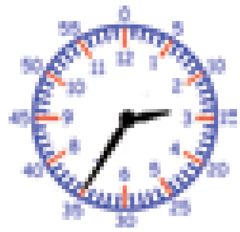
Telling the Time

Using a clock or a watch (or pictures of these dials) pupils in Year 4 will practice reading the time accurately. They will recognise that the two hands (pointers) on a clock face do different jobs – the shorter hand points to the number on the dial representing the hour; the longer hand points to the same numbers on the dial but these need to be translated into minutes.

As time passes, the hour hand moves gradually from one number to the next. Pupils need to recognise that when, for example, the hour hand points between 2 and 3, the time is **after** 2 o'clock and **before** 3 o'clock.

The minute hand will show how many minutes after 2 o'clock and, at the same time, how many minutes before 3 o'clock.

To help pupils recognise the number of minutes, you can write these around the outside of the dial until pupils become familiar with them and no longer need this guidance.



You can set tasks like the following for pupils to match these times with the clocks …

4:53	Nine minutes past seven
7:00	Seven minutes to five
2:40	Twenty minutes to three

… or you can present just one set of three times and pupils have to write the other two sets.

Note that two digits are always used to represent the minutes: show pupils that "*five past nine*" is written as 9:05 not 9:5 Another key feature to make explicit is that each hour is sixty minutes. So when the minute hand points to 3, 6 or 9, it shows the quarter hours at 15, 30 or 45 minutes past the hour. These times are usually referred to as "quarter past" the hour, "half past" the hour and "quarter to" the next hour. Pupils will become familiar with the idea that, in everyday speech, the method of describing the time changes when the minute hand passes 6, the half hour. Times after the minute hand has passed 6 are referred to as "to" the next hour. So 5:40 is often read as "twenty to six". Guide pupils to recognise that the number of minutes past the hour and the number of minutes to the next hour together sum to 60. Include the quarter hours when you ask pupils to match the different formats for saying the time.

The clock face shows 12 hours, so the 24 hour day is divided into two parts – 12 hours in the morning, measuring from midnight to midday, and 12 hours in the afternoon, midday to midnight. Midday is often referred to as "noon". To know whether a time is in the morning or the afternoon, the written time must include "am" or "pm" to show whether it is before noon (*ante meridian*) or after noon (*post meridian*). The word meridian referring to time means "the middle of the day".

10:30 am | 6:13 pm | 3:10 am | 10:30 pm | 6:10 am | 10:03 am | 3:10 am | 3:06 pm

Pupils will learn to order a list of different times, like these, into a correct sequence.

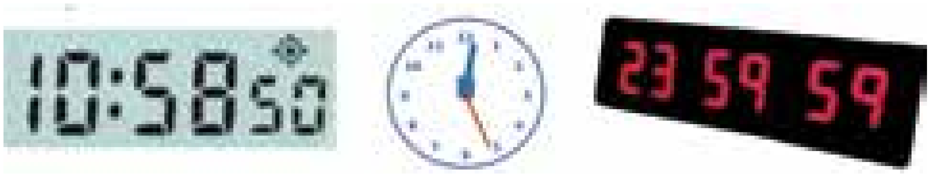
You may like to challenge Year 5 pupils by asking them to tell the time from a clock face which does not have the numerals marked on them. Each group should agree on the times.

Digital clocks and digital wristwatches display the time without using a clock face or pointers. They simply give the time using numerals. Year 5 pupils may already be familiar with digital clock displays because these are now common on mobile phones.

Many digital clocks and time displays on phone screens show 24-hour clocks. They don't show "*am*" or "*pm*" because, after noon, the counting of the day's hours continues. After 12:59 the 24-hour screens show 13:00 Then they continue counting up to midnight when, after 23:59, they continue with 00:00 On digital clocks and phones the settings allow you to choose to use the 12-hour or the 24-hour display.

When pupils have learned to read the time on the 12-hour system, which they all need to do to be able to read a traditional analogue clock, the 24-hour system can be challenging for them. For example, 19:30 translates as 7:30 pm. The hour's digit can, at first, be confusing. Pupils will need to translate 19 hours as being 7 hours after 12 noon. You will guide them to become familiar with the 24-hour system so that they will know that 06:30 is a morning time and 16:30 is an afternoon time which is the same as "*half past four*".

A *second* of time is so small that it is not a unit that is necessary for "telling the time" unless you are recording events with great precision such as in a scientific experiment. However, *seconds* can be important when measuring intervals of time. Many clocks and watches, both analogue and digital, show the seconds passing. The analogue clock will have an additional *second* hand and the digital clock has a third two-digit number.



One of these clocks shows the time at one second before midnight, one shows 1 min 10 secs before 11 o'clock and the other shows the time at 26 seconds after midday. *Which is which?*

Pupils in Year 6 will meet *seconds* when timing athletics sports events like the 60m race or the 100m race. You can link the mathematics lesson with sports by timing how long pupils take to run 60m or other distances. Further practice of using seconds can be given by asking pairs of pupils to time how long it takes to

- count from 1 to 100
- count backwards from 100 to zero
- say the alphabet from A to Z

but this activity would require the use of clocks and watches. An alternative is to ask pupils to close their eyes and raise their hand when they think that 20 seconds (or 30 seconds or 1 minute) has passed. You call out "START" and then call out when the time has reached: "20 SECONDS".

When measuring time in seconds it is difficult to be accurate because seconds pass so quickly. To measure short periods of time accurately you need a stop-watch. To time a 60m race the stop-clock is started at the beginning of the race and stopped when the first person reaches the end of the distance. The stopping of the clock "freezes" the time on the screen so that it can be read. Stop-clocks can be analogue or digital. Most mobile phones have a stop-clock facility.

Calculating periods of time

Measuring short periods of time is easy to do when you have a stop-watch. The stop-watch does it for you! Measuring longer periods of time requires a calculation. When Year 4 pupils are familiar with telling the time in the several different ways that time is displayed, they will be able to calculate simple time intervals and solve problems which involve short or long periods of time. You will help them to know, for example, that

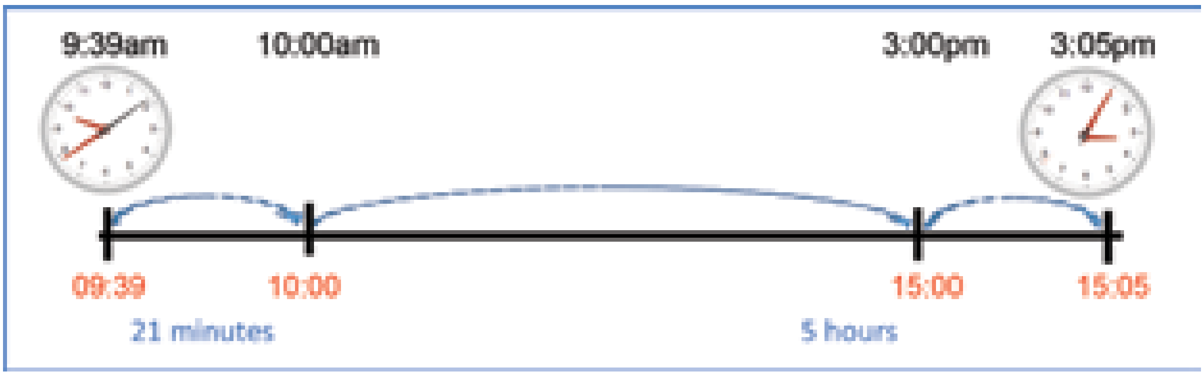
- from 9:15 am to 9:40 am is a period of 25 minutes;
- from 9:15 am to 11:15 am is a period of 2 hours;
- from 9:15 am to 2:15 pm is a period of 5 hours.

A time-line will be an essential aid to help pupils develop the concepts associated with these calculations. You will need to match the level of challenge of such questions to the pupils' abilities to read and understand the times. As pupils become more proficient at measuring time, you will develop the questions so that, by Year 5, they will be able to calculate, for example, the time period from 9:39am to 3:05pm.

The key skill that pupils need to learn is to calculate

1. how many minutes to reach the next hour,
2. the interval between the hours,
3. the extra minutes to reach the final time and
4. add the three times together.

Pupils need to do this with any of the daily time systems: analogue *clock face*; *12-hour* digital and spoken times; and *24-hour digital times*. This strategy is illustrated below on a time line. (It is not drawn to scale here.) For able pupils these four steps will become a mental calculation.



Year 6 pupils will be able to cope with problems where there is a mixture of time units. They will calculate, for example, how long it is from "quarter past nine" on Tuesday to 15:15 on Thursday. This ability will be further developed by the use of timetables. Pupils will learn how to find a time of departure and the expected time of arrival from a printed timetable to calculate the length of time of a journey. They should also be able to calculate at what time a journey should start in order to reach a destination by a desired time. *(A selection of printed timetables for trains or for plane journeys will be another group of tools that you need to collect for your mathematics teacher's toolbox!)*

Think

Can you explain why the hour hand on a watch or clock does not point directly at an hour number for more than a few seconds?

Watch

Watch the video clip MM10V5 on your phone. As you watch, think about what advantages there are in using the time line to calculate an interval of time.

Reflect

TV programmes Passenger train services Airline schedules Calendars of events

How can you collect some timetables to provide realistic questions about time intervals?

Work with your partner in school

Ask your partner to draw a clock face showing the time 1 pm. Now ask him/her to draw a second clock face showing the time 9000 seconds later. *What timetables can you collect to provide for realistic questions about time intervals?* Now it's your turn: write down the time now. Write down what the time will be in 3000 minutes from now. *Can your partner confirm your answer?*

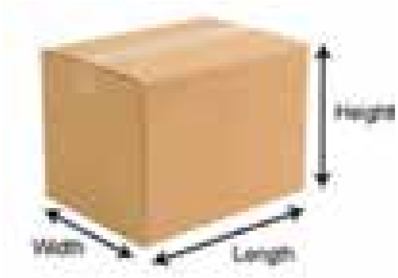
Both of you agree on the date and time 3000 hours from now.

> *Which of these problems could be tackled by groups of Year 6 pupils?*
>
> *Invent some similar questions for a future lesson on time intervals.*

For other lessons to teach about Measuring time intervals look at the TDP Lesson Plans for Year 6 Week 28. The following Lesson Plans for teaching about Time will also be of help.

P4	**Telling the time**		**Calculating periods of time**	
	Week 8	Day 1 Telling the time Day 2 Minutes “to” and “past” the hour Day 3 Digital time Day 5 am and pm	Week 8	Day 4 Changing units of time
	Week 19	Day 4 24-hour clock Day 5 Digital time.	Week 20	Day 1 A calendar Day 2 Time number lines Day 3 Time problems Day 4 A train timetable Day 5 Multiplication time problems.

Summary of Module 10



You have seen that to measure the size of an object: it's Length, Area, Volume, Capacity and Weight, each property has its own unit of measurement: metre, square metre, cubic metre, litre, gram.

Children soon appreciate that one standard unit of measurement is not suitable for every size of object: you cannot sensibly use litres to measure a small amount of medicine; you cannot sensibly use grams to measure a lorry-load of sand; and you cannot sensibly use seconds to measure your age (although it may be of advantage for learning mathematics if Year 6 children try to answer the question of whether they have yet lived a million seconds!). Each of these units of measurement has variations to cope with small, medium and large sizes. The international units of measure, except for Time, are all based upon multiples of 10 and form the decimal metric system. The most used measures are kilo-s, centi-s and milli-s.
1000 units of measure form a kilo-unit; $^{1}/_{100}$ of a unit is a centi-unit; $^{1}/_{1000}$ of a unit is a milli-unit.

The relationship between the different sizes of the standard units should be a feature of work with Year 6 pupils. They will explore the relationship between a millimetre, a metre and kilometre and discover that this is the same relationship between a milligram, gram and kilogram. Although milligrams are only used for very, very small amounts – such as scientists measuring tiny quantities of chemicals needed for a tablet of medicine (like 200mg of paracetamol in a tablet to ease pain) – pupils can calculate how many milligrams make 1 kilogram. *How many millimetres are the same as 1 kilometre*?

In this module you have been given some guidance on the progression from simple to advanced ideas for teaching measurement in P4-6. Because measurement concerns real everyday life, such as buying drinking water, the 23kg weight restriction on airport baggage and measuring material to make clothes, you should use lots of real-life examples so that pupils will better understand the uses of mathematics and be better prepared for their future lives - as well as being well-prepared for examinations in mathematics. Some examples of good teaching have been suggested. Below are some more activities.

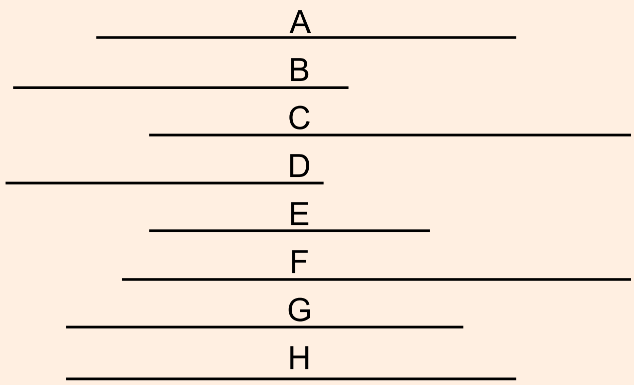
Ideas to try in the classroom

When you try the following activities, write a note about your experiences: what worked well; what the challenges were; what you did to overcome any challenges and what difference the activity made in your classroom. Bring your notes to the next cluster meeting.

Try in the classroom 1

Topic: Measuring to the accuracy of millimetres

Give each group of pupils a set of lines which are all similar in length. Pupils will measure each line as accurately as possible using centimetres and millimetres (or centimetres using decimals), then arrange the letters in order of size, shortest first.



Try in the classroom 2

Topic : Estimating, measuring and adding / subtracting lengths

Duration: 45 minutes

Objectives: By the end of the lesson, pupils will be able to

- estimate distances;
- add and subtract distances.

Teaching aids: metre rule, tape measure

Previous knowledge: pupils know how to measure the length of a their desk

Step 1 Discuss what it means to make an estimate. Let pupils see a metre and a centimetre. Agree which unit of measure is appropriate for estimating the length of the chalk board. Agree with the class a suitable estimate for the length of the chalk board.

Step 2 Show pupils how to measure the length of the board as accurately as possible in metres using a tape measure or a metre stick. Discuss how close the estimate was.

Repeat Steps 1 and 2 to estimate, then measure, the height of one pupil in centimetres.

Step 3 Pupils now choose at least 6 objects to estimate. They prepare a table for recording the estimates and measurements and write their objects in the table. They should agree which unit is the most appropriate and write this in the third column. They then write an estimate for each item.

	Object chosen	cm or metre	Estimate	Measxurement	Difference
1	Door height				
2	Height of Ali				
3	Length of room				
4	Width of desk				
5	Length of window				
6	Width of window				

Step 4 Pupils now measure their chosen items. They record their results and calculate the differences between their estimate and their measurement

Step 5 Pupils are now given a list of estimates and measurements that have been already made in metres and centimetres. They are required to calculate the differences for these measurements (the mixture of metres and centimetres will require you to demonstrate how to use 1m = 100cm to make the calculation possible.

	Object chosen	cm and metre	Estimate	Measxurement	Difference
1	Door height		1 metre 20cm	96cm	
2	Height of Nafisa		1 metre 30cm	97cm	
3	Length of wall		5m	4m 26cm	
4	Verandan width		2m 50cm	3m 10cm	
5	Window height		1m 20cm	1m 22cm	
6	Width of folder		30cm	25cm 5mm	

Step 6 Ask pupils to calculate the length of a row of 3 classrooms which are each 8m 35cm long.

Try in the classroom 3

Topic : Telling the time, "past" and "to" the hour

Duration: 45 minutes

Objectives: By the end of the lesson, pupils will be able to

- know the units used to measure time;
- tell the time from an analogue clock face, using "minutes past the hour".

Teaching aids: cardboard clock face to show hours and minutes; a real clock;
ESSPIN Numeracy lesson plans P4 Week 8 Days 1 and 2
Macmillan New Primary Mathematics Book 4 pp 137-142
New Method Mathematics Book 4 pp 183-193

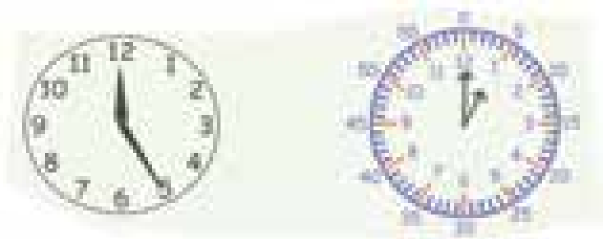
Step 1 Introduce the topic of Measuring time using units of time.
What is the smallest unit of time that pupils know?.

Step 2 Write the following list on the board and ask each group of pupils to help you fill in the missing numbers.

_____ seconds make 1 minute	_____ weeks make 1 month
_____ minutes make 1 hour	_____ months make 1 year
_____ hours make 1 day	_____ weeks make 1 year
_____ days make 1 week	_____ days make 1 year

You will need to explain that the months are not all the same length. Some are 31 days, some are 30 days and one month (February) is usually 28 days but, in Leap Years it has 29 days. This is because the Moon's movement around the Earth is not an exact number of days. But each month is at least 4 weeks. The numbers of the list confirms that *seconds* are the smallest unit of time.

Step 3 Discuss your demonstration clock with the class. Ensure that the pupils can recognise the hour hand and the minute hand. Help them to understand that the same dial is used to show 12 hours and 60 minutes by drawing the minutes around the outside of the dial of hours.



Step 4 Turn the clock hands to create times for pupils to read. Start asking pupils to read the time with the hour hand pointing directly at the hour. Limit the first questions to "quarter past", "half past" and "quarter to" the hours. Ask pupils to read your clock times. Then change to the minute numbers. Pupils will read the times which you make.

Step 5 Change to you declaring times and asking pupils to move the hands to show the time.

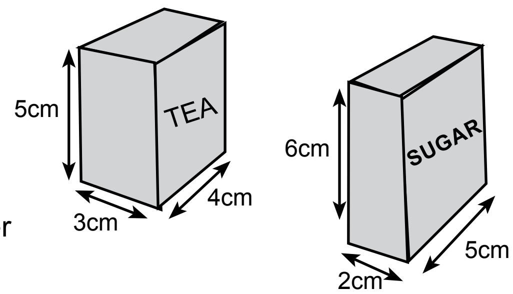
Step 6 Discuss the change in reading the time when the minute hand passes the "6". Ensure that pupils can swap times saying the same time, for example, 4:40 as "Forty minutes past four" and as "Twenty minutes to five".

Step 7 Finally, move to the reality of the hour hand moving between the hours as the minute hand rotates. Discuss the position of the hour hand at "quarter past", "half past" and "quarter to" the hours.

Experiencing change in your classroom

Once again, you are reminded to use your journal to make notes of your experiences after teaching in your classroom. It is important as a teacher that you form the habit of noticing what your pupils learn and observing if they have learnt what you had intended. Developing your ability to make informal, but informed, assessment of the lesson will allow you to plan appropriately for the next lessons and build effectively on the previous lesson. The following questions will guide you to write in the journal about your experience:

- Which activities did you try out in your classroom?
- Which ones went well? Why?
- Which ones were less successful? Why?
- What changes could you make to ensure better progress by the pupils?



- Were you able to bring some appropriate teaching aids to the Measurement lessons to exploit the everyday aspects of measuring lengths, volumes, weights and time? Have you used real examples to engage the pupils with mathematics; for example, asking pupils which packet will weigh more - the tea or the sugar?

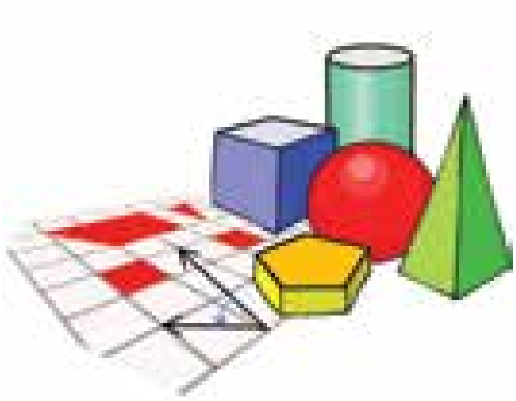
Suggestion for the next cluster meeting

Don't forget that adapting your teaching skills in the light of this guidance will not always be immediately productive, particularly if this requires changes in the pupils' classroom practice. Persevere with developing your new skills and use your deeper understanding of the underlying mathematics principles to be a better teacher. You may have some thoughts on this to share with your colleagues in the next cluster meeting.

Write in the space below any points which you would like to discuss in the next cluster meeting. The topics can include a comment, a challenge or an experience that you would like to share with your fellow teachers.

Module 11: Geometry

Module 11:
Geometry



Geometry is the mathematics of Shape. It includes the studyof 3-dimensional objects (often referred to as solids) and the learning about the properties of 2-dimensional shapes (often called plane shapes because they are flat and lie on a flat surface). Geometry also includes the study of position and direction. A key feature which underpins all these areas of teaching Geometry is the learning about angles and lines.

Pupils will be familiar with many shapes but you will need to teach them to recognise the many characteristics or properties that they have. For example, a lunch box and a match box are both cuboids, both with six faces and all these have square corners in all three dimensions. The Moon, a football and a table tennis ball are spheres, each with only one surface. A photograph, a chalk board and a rectangular floor mat all have two pairs of opposite sides which are equal in length Every triangle, whether in the classroom roof timbers or in a bicycle frame, has three sides and three angles.

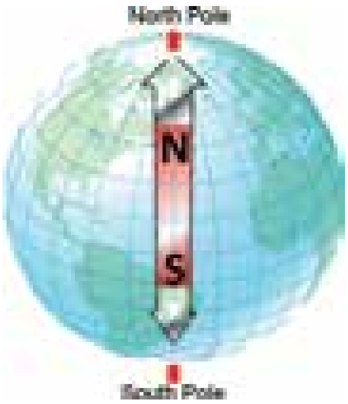
This module will guide you to effectively teach about the shapes that we have mentioned here. Pupils will learn about the shapes the objects have and the properties of the shapes, not the objects themselves. So the following sections for Geometry in Years 4 to 6 focus on:

Section 1	Position and Direction	(mainly Year 4)
Section 2	Lines and Angles	(Years 4,5 and 6)
Section 3	2-D *Plane shapes*	(Years 4,5 and 6)
Section 4	3-D *Solids*	(mainly Year 5)
Section 5	Symmetry	(Years 4, 5 and 6)

Section 1:
Position and Direction

This section on Position and Direction is introduced by thinking about travelling in Nigeria but the same principals are important for mathematics to describe the co-ordinate grid system which pupils will meet in Years 5 and 6 and which is an important part of advanced mathematics.

The cardinal points are the directions North, East, South and West. These describe the directions of other people or objects relative to where you are standing. The directions of North and South are fixed because these show the directions to the "top" and "bottom" of the Earth on which we all live.



The directions East and West are the directions to your right and to your left, respectively, when you are facing North. Most pupils in the northern states of Nigeria are probably familiar with the direction of East because any teacher taking the morning prayers for Muslim students will expect to face East towards the Kaaba.

Help the pupils to have a sense of the cardinal points by discussing a map of Nigeria. Identify some states which are in the north of Nigeria.

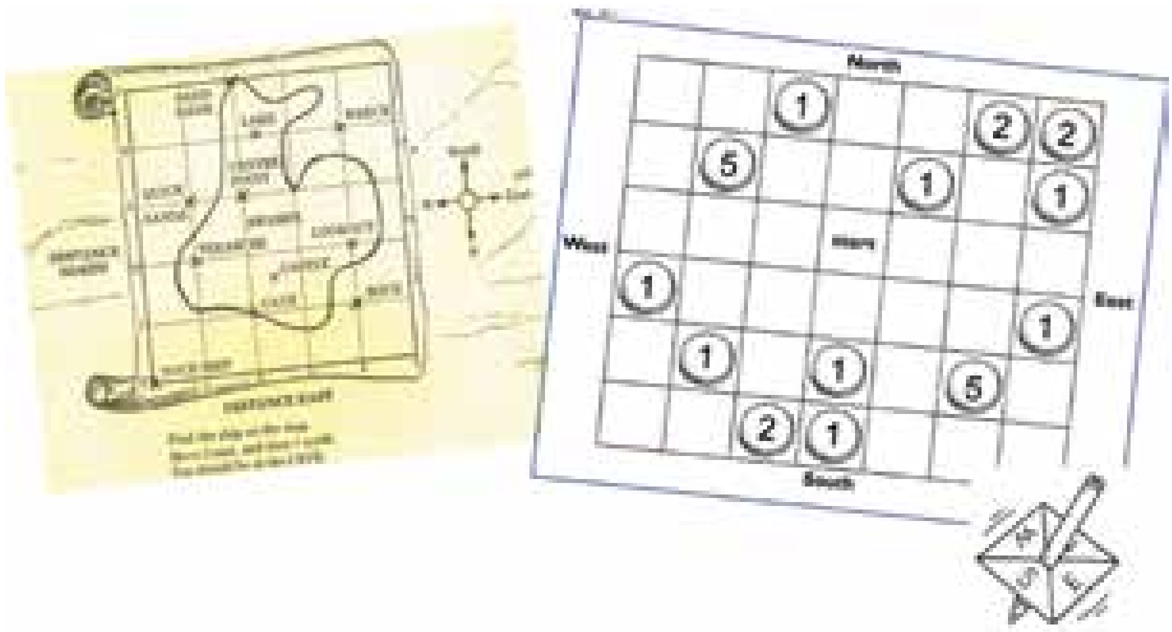
IS BENIN CITY WEST OR EAST OF HERE ?

NAME TWO STATES IN THE WEST

WHICH CITIES ARE IN THE EAST ?

Pupils also need to have a sense of the directions being relative to where you are positioned. The states which are known as Northern Nigeria or Eastern Nigeria describe their position relative to the central Federal Capital Territory and Abuja. Zamfara is a state in the north of Nigeria. But if you are in Sokoto, Zamfara is south of you. Nigeria is in the east if you are in Ghana but in the west if you are in Cameroon.

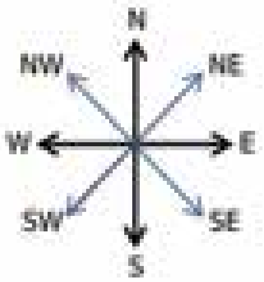
To enable pupils to be familiar with the cardinal points, you can use a variety of instructions to direct pupils. In Game 1, direct pupils to walk from their room using instructions such as “Turn left”, “Walk forward ten paces”, “Turn right”, “Walk four steps backwards”, “Turn right again”, “Walk forward five paces”, … Pupils who move in the wrong way at any stage are out of the game. In Game 2, establish a starting place and which direction is North; then direct pupils using the compass directions: “Take 5 steps north”, “Go 10 paces east”, “Now step 4 paces south”, “Go 2 steps north”, … These games are, necessarily, played outside of the classroom. An alternative activity is to ask children to describe a journey using only the compass directions. This can be done in real life, for example by directing someone from the school gate to the classroom. It can also be done by using a map of an imagined island or a map of squares.

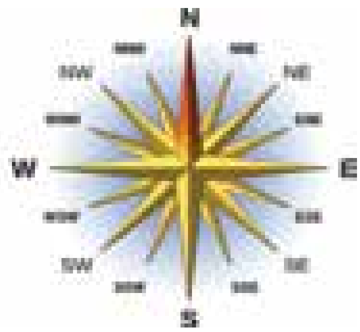


For the map of squares on the right, make a spinner with the four compass directions. Each player starts with a counter on the middle square. Each player take turns to spin the spinner and then moves their counter one square in the direction indicated. The player scores the point if their counter lands on a number. The winner is the first player to collect 8 points.

Pupils will be familiar with the direction moved by the hands on a clock and so you can introduce the ideas of clockwise and anticlockwise to define a direction of turning. Ask a pupil to stand facing South. He should then make a quarter turn clockwise. In what direction is he now facing? Ask a second pupil to face West. She should then turn halfway round anticlockwise: In what direction is she now facing?

Pupils can pick three of your home-made cards to receive instructions like these. Their group members will check if they make the correct turns. Pupils will understand through these games that a half-turn clockwise gives the same outcome as a half-turn anticlockwise; a quarter turn anticlockwise has the same result as a three-quarter turn clockwise.





The direction of North can always be found by a magnetic compass. But the four cardinal points do not give enough accuracy for most navigational tasks. To give more detail, four more points are given to describe the directions between North, East, South and West. NE (North-East) is the direction halfway between North and East. SE (South-East) is the direction halfway between East and South; and so on. Year 6 pupils will be interested to know that, in the days before computers could be used to aid navigation, ships at sea needed more accurate directions. Sailors used directions between the eight points, giving another 8 directions. North North East (NNE) is the direction between North and North East. East North East is the direction between North East and East, and so on.

Think

From which direction does the Harmattan wind blow? Which direction does it blow towards?

Watch

Watch the video clip M11V4 on your phone. As you watch, think about the teacher's classroom organisation for this lesson starter.

Reflect

- How might the teacher have continued the lesson to give pupils some practical experience of rotating clockwise or anticlockwise to change direction?
- If you have a pupil who has learning difficulties or is visually challenged, how would you help the child to learn about directions and the cardinal points?

Work with your partner in school

With your partner, prepare a sketch map of your village/school environment and locate several buildings. Prepare a list of questions that will allow pupils to locate different places…

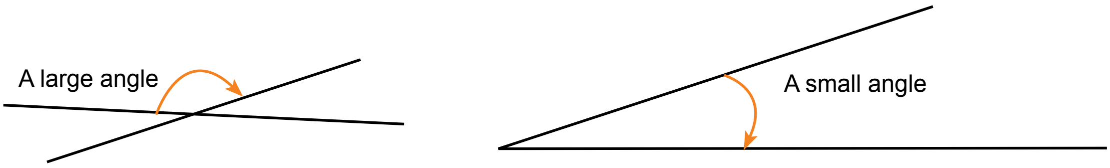
Whose house is East of the market? In what direction is the Health Centre from the school?

…and questions that will allow you to use journeys which follow directions:

From the school go South West for 100m; turn North and walk 20m. Go west for 50m. What place have you reached?

Section 2:
Lines and Angles

An angle is formed when two lines intersect. Pupils, and teachers too, will inevitable see an angle as the “corner” of a shape. It’s naturally how you will recognise the angle.



However, pupils will have problems with understanding angles and learning geometry if they are introduced to *angle* as “*shape*”. Angles are not shapes. Angles are “*turns*”. Angles are measured by how much you need to turn from one line to the other. The above diagrams hint at the possible confusion because the angle on the right looks to be a larger “shape”. But the amount of turn in the angle on the right is much smaller than the amount of turn for the angle indicated in the diagram on the left. To give pupils a good foundation, you are encouraged to present angles in Year 4 by first introducing the ideas about turning associated with the Compass Directions treated in Section 1. Use the Lesson Plans for Year 4, Week 6 before treating Lines and Angles. As you will see in the Lesson Plans for this week, you will need to integrate Sections 1, 2 and 3 carefully when you teach Geometry in Year 4. Let us now look in detail how to introduce angles.

You know from thinking about the cardinal points N, E, S and W and from teaching pupils about telling the time that pupils will be familiar with the idea of turning around in a complete circle. They will understand half-turns and quarter turns. They will understand clockwise and anticlockwise directions. This is the most appropriate starting point for teaching about angles.

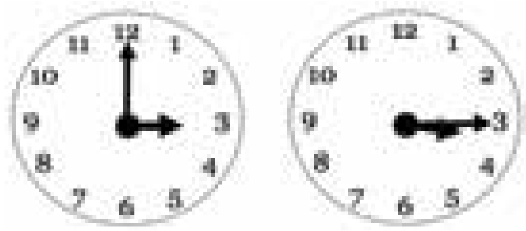
In ancient Babylonian times, people recognised that the position of the stars in the night sky changed a little every day. They saw that the stars gradually rotated. People realised that the stars, and also the seasons of the year, came round again after 360 days. We now know that the yearly rotation is more accurately 365¼ days but we still use the Babylonian idea of measuring a complete rotation with 360 degrees. (The symbol $^{\circ}$ is used for degrees). This means that turning through half a circle is a turn of 180°. Making a quarter turn is a rotation of 90°. Turning 270° is three-quarters of a whole rotation. Ensure that pupils understand that turning around twice, facing the same direction that they started from, is a rotation of 720°.

If you start facing south and turn 90º clockwise, what direction will you now face?

When you are facing the chalk board and turn 180º, which way will you now be facing?

Face the door.. Turn 270º anti-clockwise.. How many degrees must you turn to face the door again?

You can use your model clock face to ask more questions about rotating.



How many degrees has the minute hand rotated between three o'clock and quarter past three?

How many degrees has the minute hand rotated between two o'clock and three o'clock?

A range of similar questions will establish the notion that to measure the size of an angle is to measure how much turning has occurred. *Can you answer these questions?*

Angles on a clock

A full turn or one circle is 360º

360º

... so % of a revolution is 90º

90º

... and $^1/_2$ a revolution (which looks like a straight line) is 180º

180º

1 What is the angle thurned by the minute hand in 20 minutes?

2 What is the angle thurned by the minute hand in 10 minutes?

3 What is the angle thurned by the hour hand?

4 How many degrees does the hour hand turn in 9 hours

5 How many degrees does the hour hand turn in 18 hours

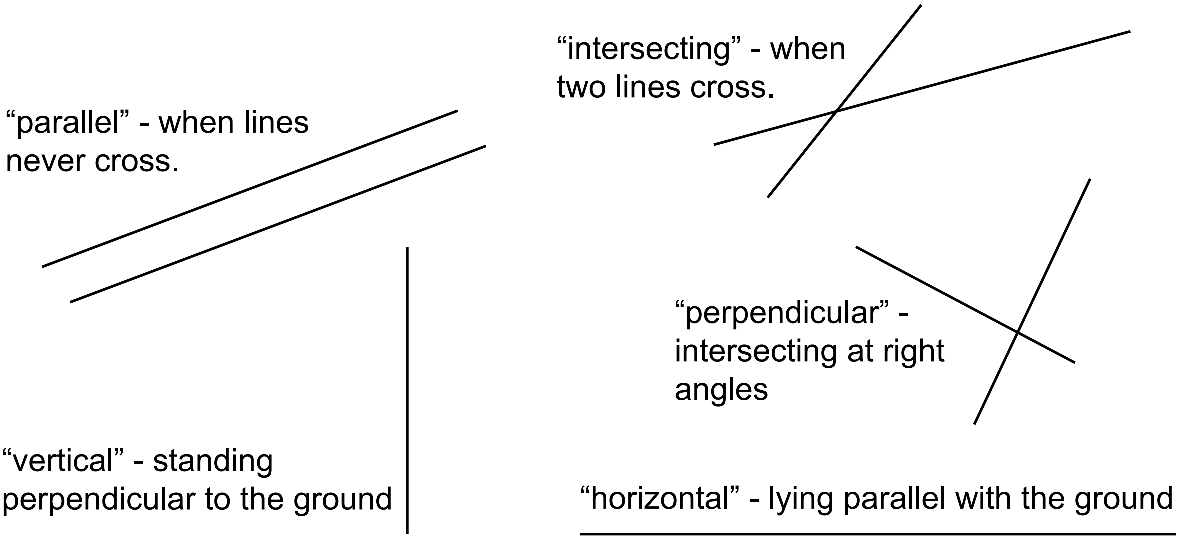
6 Calculate the number of degrees each watch hand turns in $^1/_2$ hour

7 The three hands on a watch are all in line at 12ºº noon. What is the next time when they are all in line?

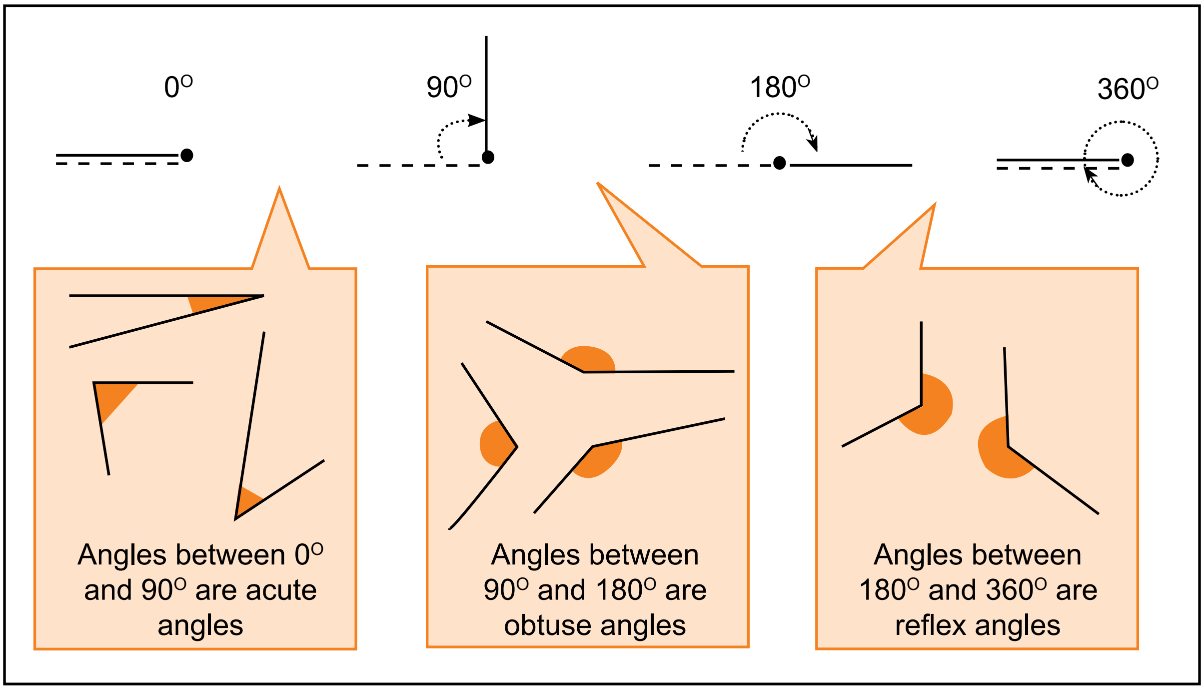
Question 7 is challenge!

From these questions you can establish that a complete turn of 360° is the same as 0°; that a half-turn of 180° is like a straight line; and that a quarter turn of 90° is like a square corner. These ideas are so important for working with angles that the 90° angle has its own name – the **right angle**. 180° is the same as two right angles. 360° is the same as four right angles.

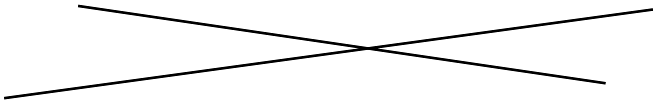
Two sticks will also help you to discuss angles with your class. These have the advantage that you can ensure that pupils understand the use of words like …



The angles associated with a quarter turn, a half turn and a whole turn give recognition to types of angles which are between the four distinct stages of a rotation.

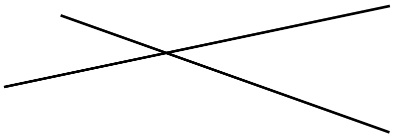


The vocabulary of words associated with angles gives you the language to teach pupils about the mathematics of shapes that you will study in the remainder of this Module. But before you complete this Year 5 work, we will spend a little more time with your two sticks.

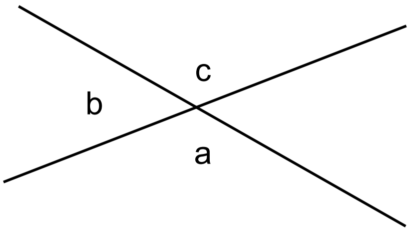


Place your two sticks to illustrate an intersection. You will have formed two acute angles and two obtuse angles. As you rotate one stick on top of the other, what happens to the angles?

Can you see that the two acute angles could be equal to one-another? Does it look like the two obtuse angles are equal to one-another?



- Describe what happens when you rotate one stick: do the acute angles get smaller? What happens to the acute angles as the two sticks become parallel?
- What happens when the acute angles get bigger? When do the acute angles become obtuse angles?

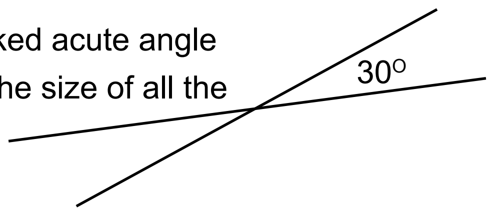


Three of the angles on the intersecting sticks on the left have been labelled *a*, *b* and *c.*

b and *c* together are the same as 180° because they form a straight line.
b and *a* together are the same as 180° because they also form another straight line.

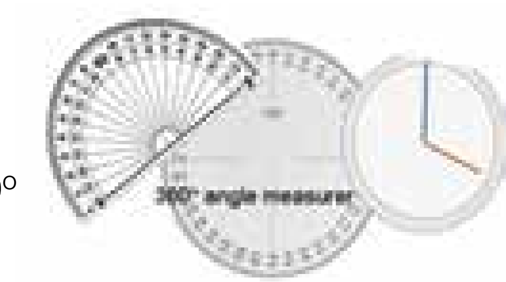
What does this tell you about angles c and a?

Can you use similar logic to prove that the unmarked acute angle is equal to angle *b*? Use this knowledge to know the size of all the angles at this intersection where one angle is 30°



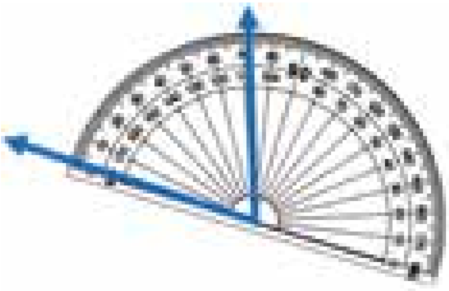
Pupils in Year 6 will learn how to measure and how to draw angles accurately using a protractor or an angle measurer.

common protractor
with angles up to 180°

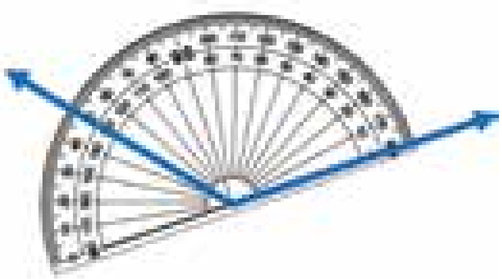


angle finder
with no angles

This is a practical skill, not one that can be taught by reading about it. Protractors are designed so that an angle can be measured from left to right or from right to left -- that is clockwise or anticlockwise. To allow this facility, the protractor has two sets of scales, one on the outside for measuring in one direction and one on the inside for measuring in the opposite direction. However, this design which is intended to make the protractor convenient to use, confuses many children at first. You will need to guide pupils to place the centre of the protractor on the intersection point of the angle being measured. At the same time, the zero line is placed over one of the angle's lines. Depending upon which zero line is placed over the first angle line, the scale is chosen to locate the reading of the second angle line.



Measuring an acute angle of 72° from the zero on the left using the outer scale.



Measuring an obtuse angle of 130° from the zero on the right using the inner scale.

Think

Why could many pupils have the idea that angle is a shape not a rotation?

Watch

Watch the video clip M11V2 on your phone. As you watch the three extracts of a lesson, can you identify the characteristics of the three different episodes?

Reflect

- How did the teacher start the lesson?
- How did she help pupils to have a mental picture of the angles discussed?
- What equipment was necessary for measuring angles?
- Did the teacher include any reference to angle as rotation?

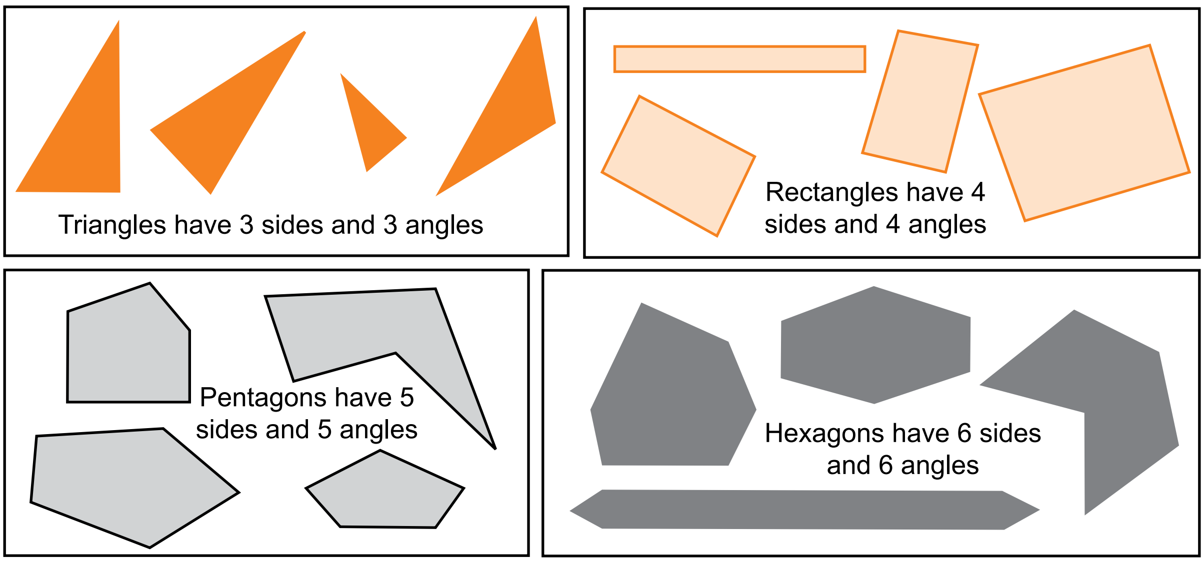
Work with your partner in school

With your partner, prepare a list of five clock times when the hour hand and the minute hand have a right angle between them. (Remember that the hour hand moves as the minute hand rotates, so the hands are not perpendicular when the time is 3:30)

List five times of the day when the hour hand and the minute hand make an angle of 180°.
Can you find five times of the day when the angle between the clock hands is an obtuse angle of 120°? …or a reflex angle of 240° ?
Consider how a piece of paper might help to make this activity accessible for year 5 pupils.

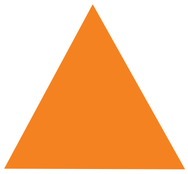
Section 3:
Plain Shapes

Pupils in Years 1 and 2 were able to sort shapes into groups -- identifying squares, rectangles, circles and triangles. They were able to recognise that some shapes have curved edges and some have straight edges; that some shapes have "square corners" and others do not. In year 4 you will help pupils to recognise that the 2-dimensional shapes with straight edges and angled corners are named according to the number of sides and the number of angles that they have.



7-sided shapes are called heptagons. Octagons each have 8 sides. Nonagons are 9-sided and decagons have 10 sides. Each polygon has its own name according to its number of angles.

Plane shapes like those above, made with straight lines, are called polygons – the word means “many angled”. Circles, semi-circles and ellipses are plane shapes but they are not polygons.



When a triangle has 3 sides of equal length, its angles are also all equal. This is a perfectly regular shape and is called an equilateral triangle. The word **equilateral** means *“equal sided”*.

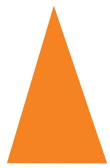
All rectangles have equal angles too but they may have sides of different lengths. What special name is given to a rectangle which has all its sides equal ?

When a polygon has equal sides and equal angles it is referred to as a “**regular**” polygon:

Year 5 pupils will learn that the types of angles which form the corners of the shapes define the shapes more precisely. You will need to teach pupils the vocabulary which enables them to recognise and describe these properties. Do this through discussing the shapes and asking pupils to write their own notes and to explain to their group, not by giving pupils definitions to copy.

A triangle that has 2 sides with the same length is an **isosceles** triangle.
Because two sides are the same length, two of the angles are the same size.
Which two angles?

Help the pupils to recognise that an isosceles triangle can contain an obtuse angle (as on the left) or only acute angles (as on the right)...



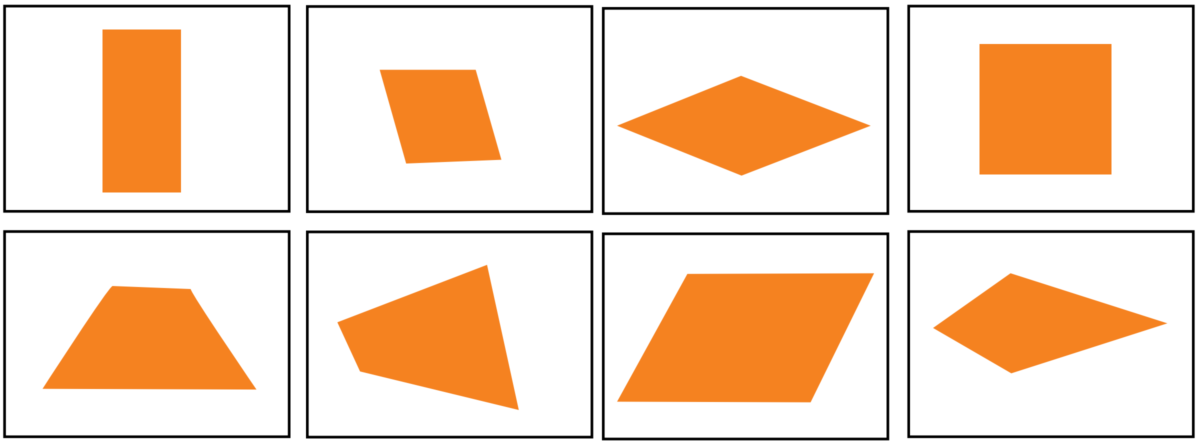
…. and help them to discover that a triangle cannot have two equal angles that are obtuse. *How will you help them to know this?*

Pupils should be able to recognise an isosceles triangle in any orientation. The equal sides are usually shown by making a mark across them.

A triangle with no equal sides and, therefore, no equal angles is called a **scalene** triangle.

Younger pupils know that rectangles have four right-angles and two pairs of parallel sides but rectangles are not the only polygons with four angles. There are many other 4-sided shapes that have obtuse and acute angles. Polygons with four angles are all **quadrilaterals**; the word means “four-sided”. A scalene quadrilateral is usually called an **irregular quadrilateral**. One is included in the following diagram. A square is the only regular quadrilateral.

The diagram contains all the different types of quadrilaterals, two of them are the same shape. *Can you match the three sets of cards showing the shapes, their names and their properties ?*



square	kite	parallelogram	rhombus	rectangle	trapezium	irregular quadrilateral

• Two obtuse angles • Two acute angles • Two pairs of equal sides • Two pairs of parallel sides	• Two obtuse angles • Two acute angles • Four sides equal • Two pairs of parallel sides	• Two obtuse angles • Two acute angles • Four pairs of equal sides • No parallel sides

• Four right angles • Two pairs of parallel sides • Two pairs of equal sides	• Four right angles • Four sides equal • Two pairs of parallel sides	• No equal angles • No equal sides	• One pair of parallel lines

Think

Which statements are correct?

1. “A 2-dimensional plane shape is a flat shape.”
2. “Circles and semi-circles are plane shapes.”
3. “Circles and semi-circles are not polygons.”
4. “Triangles and quadrilaterals are polygons.”
5. “Polygons have the same number of angles as the number of their sides.”

Explore

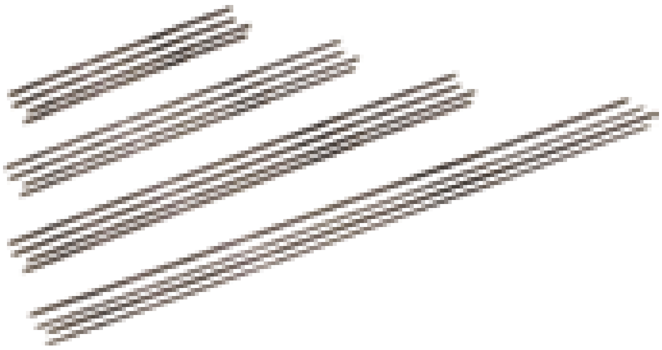
- The word ***quadrilateral*** means "four-sided". Can you draw an **equilateral quadrilateral**? *What shape could this be?*
- Can you draw a trapezium with no equal sides?
- Which shape is a parallelogram with four right-angles?
- Can a quadrilateral have a reflex angle inside?
- Can you draw a quadrilateral which has exactly three of its sides equal? *What shape could it be?*

Reflect

Is a rectangle a parallelogram?
Is a square …
… a rectangle with equal sides?
… a parallelogram with equal angles?
… a rhombus with right angles?

Work with your partner in school

Work with your partner in school to make some sticks which you can use to construct quadrilaterals.



Four each of four sizes such as 20cm, 30cm, 40cm and 60c would be good lengths so that the class can easily see the shapes you will make. (You could use corn stalks but they are easily bent.)

Plan a lesson in which you will gather the groups of pupils around one central area and you can demonstrate how to make different quadrilaterals by placing four sticks on the floor of the classroom.

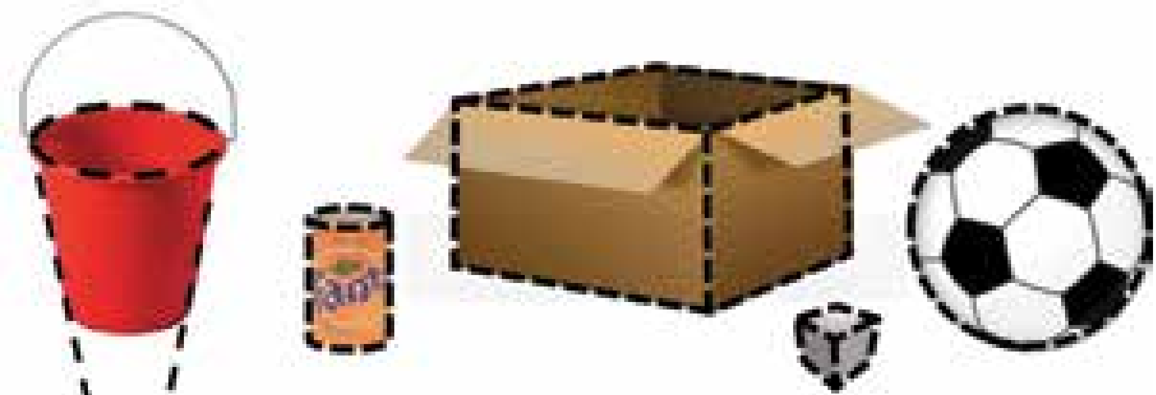
The lesson should include challenges for the pupils to use the sticks to make

- 2 different parallelograms
- 3 different squares
- an irregular quadrilateral
- 2 different kites
- a rhombus
- a trapezium
- a trapezium with two right angles

Pupils have to describe the shapes and the angles that they see at each corner.

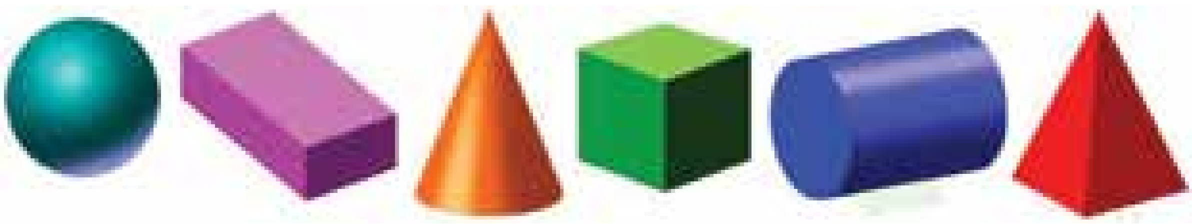
Section 4:
3-D Shapes

Pupils will be familiar with many solid objects around the home. Geometry deals with the solid shapes with surfaces that are plane shapes or are circular (such as boxes, balls, buckets, etc) and explores what other 3-dimensional solids are possible.

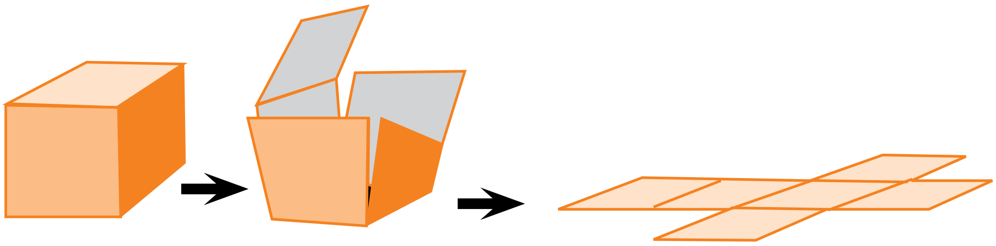


You can see that a bucket is part of a **cone** and that a box is an open **cuboid**. Your drink can is a **cylinder** and your football is a **sphere**. Your stock cube or sugar cube is, of course, a **cube**. This shape recognition will have been the feature of how pupils in Years 1-3 will have been introduced to sorting solids. In Year 4, the characteristics of a solid shape are defined by the number of faces, edges and vertices that it has. Spheres have only one surface and no edges.

A cone has one curved surface and one circular face. A cylinder has one curved surface and two circular faces. Guide pupils to recognise the features of other solids using actual objects.

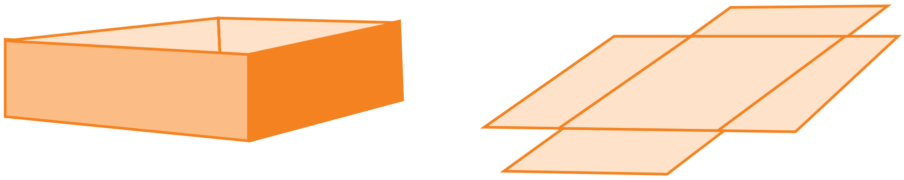


The shapes of the faces of a solid are also key features for pupils to recognise. They should recognise that a cube is a **regular cuboid** – a cuboid with each edge the same length, each angle a right-angle and each face a square. It is regular because each face is a regular polygon.



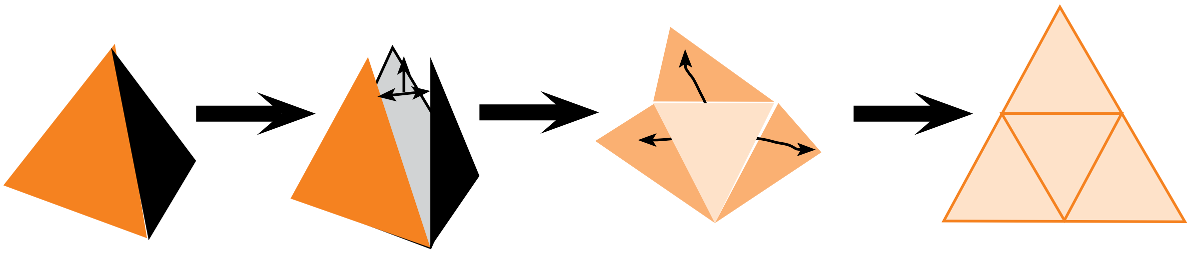
Pupils in Year 5 need to witness the cutting open of a box so that they can see how the cube has been constructed from a net of six squares. Contrast this with the flattening of an open box.

A matchbox tray would be a good example to provide:

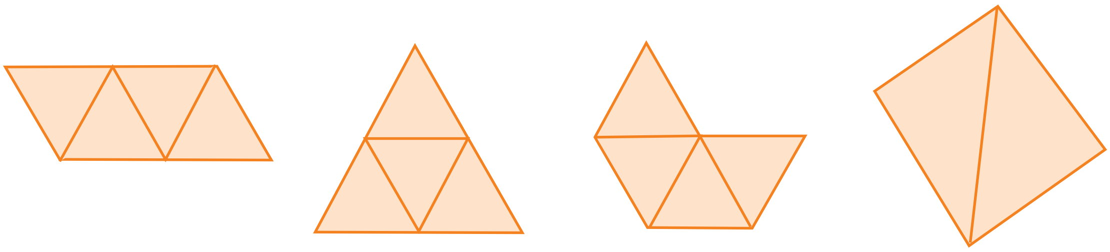


A **tetrahedron** (meaning “4 faces”) is a regular shape too. Each of its faces is an equilateral triangle. All the edges are the same length and all its angles are equal.

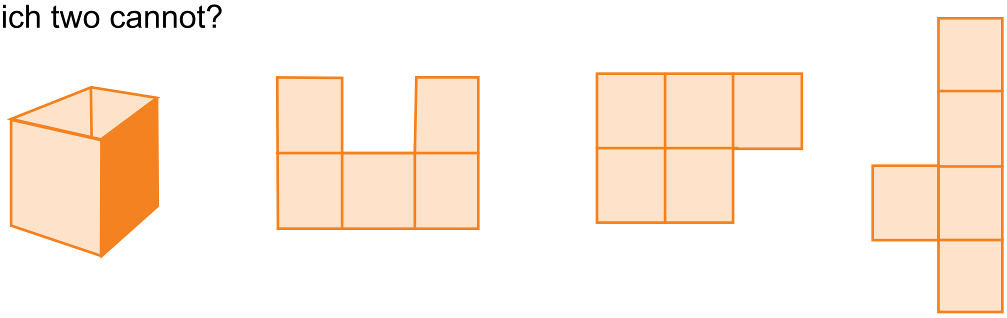
Its net is a triangle of four equilateral triangles.



Only two of the nets below will fold

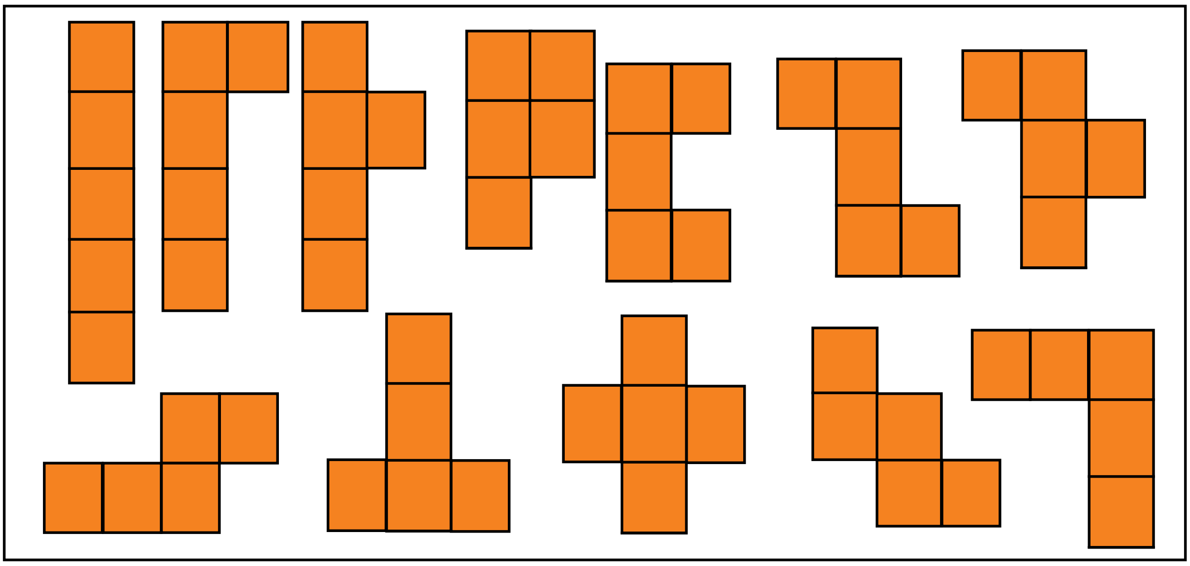


Only one of the following pentomino nets will fold to make the open box. Which two cannot?



Because 5 squares joined edge-to-edge can make several different pentominoes pupils can be challenged to find all the different arrangements of 5 squares.

It is possible to make 12 different pentominoes which are not reflections or rotations of one-another. Pupils can use small squares to find these and discover which of them will fold up to make an open box.

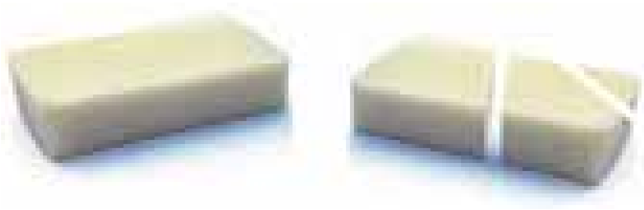


Pupils can work in groups to cut out small squares and discover these pentominoes. 4cm x 4cm squares are a good size to work with. Squared paper will make it easy for each group of pupils to record their different shapes.

Think

Can you find or think of 6 different pairs of solids which have similar shapes? Here are two examples to start with.

Explore



Cutting across a soft soap bar can produce a variety of shapes. With these two cuts you can see a trapezium, a pentagon and a triangle on the top … a rectangular cross-section … and a small pyramid.

What other shapes could a straight cut produce?

Reflect

Thinking of the properties of solids that your pupils will come to know practically, how can they record this information if they will find sketching 3-D objects difficult to do?

Work with your partner in school

Work with your partner in school to discover how many different cuboids you can make using 6 match boxes.

If you are able to collect together about 30 empty matchboxes, you would have enough for a lesson in which five groups of pupils could each investigate how many cuboids

can be made with 6 boxes.

If pupils will find it difficult to record their results by drawing 3-dimensional sketches, they can record their results in a table like this … or by measuring the actual length, width and height of their cuboids in centimetres.

	Length	Width	height
1.	6	1	1
2.	3	1	1
3.	1	1	2
		2	3

If you have a smart phone, you could also take some pictures of their cuboids as a record of their work.

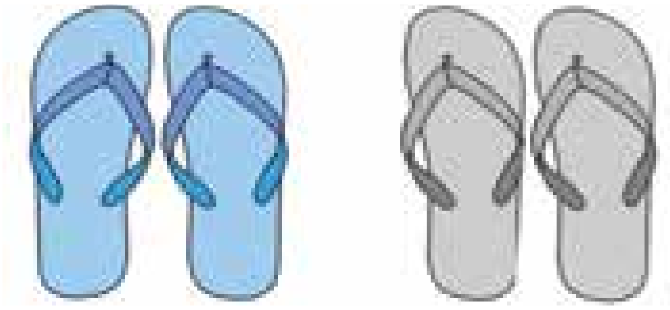
Section 5: Symmetry

Pupils in the early years of Primary School readily recognise the idea of things being *upright* and they grow to understand the idea of things being *level*. So, by Year 4, they can understand the use of the words “vertical” and “horizontal” which you met in Section 2. However, pupils up to the age of about seven do not easily distinguish left from right with any certainty. This is not surprising because describing something as left or right depends upon the point of view of the observer. Moreover, if you look in a mirror, the right ear of the person you see in your reflection is your left ear! Creating symmetrical shapes helps pupils to know left from right.

The growth of spatial concepts is a key aspect of learning Geometry. This development includes the idea of balance, with the left and right of an object being perceived as identical, except for position. Nature tends to develop this symmetry so that our bodies are roughly symmetrical. Most living things are symmetrical - think of animals and living objects such as leaves and fruit.

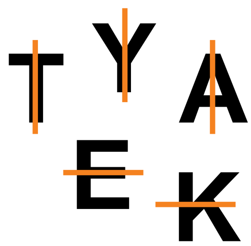
A symmetrical design naturally leads to the idea of equal halves and the recognition of cutting along a line of symmetry which separates the two halves. The Year 4 work will focus on the symmetrical halves of plane figures. This extends the study of the properties of plane shapes that you looked at in Section 3. The symmetry of 3-dimensional solids is a feature of secondary school mathematics but the basic notions of symmetry in solids is also relevant for Year 6.

Pupils' perceptions of symmetry and the mathematical idea of symmetry are both based upon the perception of opposite halves. This can be illustrated by looking at the blue and grey Flip-flops. Which is the symmetrical pair?



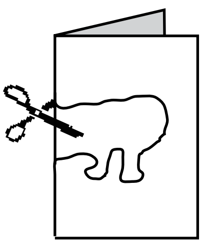
The blue pair is symmetrical. Although the two grey flip flops are exactly the same, the left shoe is not the opposite of the right shoe. So the idea of two halves being

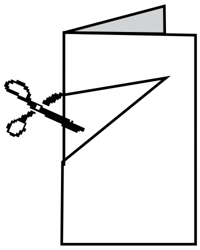
the same is not sufficient for symmetry. The understanding of symmetry is confirmed by the idea of reflection. When you are judging whether a shape has symmetry, you will not be looking for the left side to be exactly the same as the right side but, rather, a mirror image of it. This will be the same for 3-D solids.



Pupils can explore this by examining the capital letters of the alphabet to find the 11 letters which have left/right symmetry. The mirror line can also be placed horizontally: pupils can discover the 9 letters which have top/bottom symmetry. Four of the letters are in both groups, each having two lines of symmetry. *Which are they?*

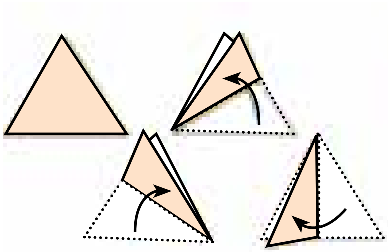
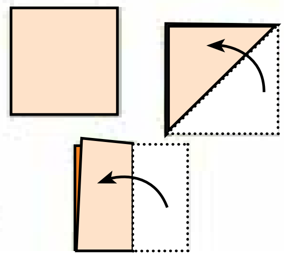
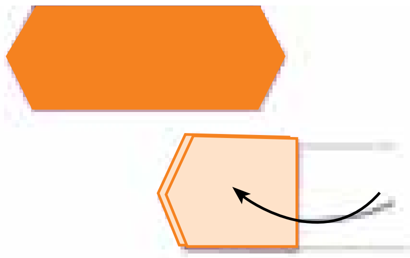
Pupils can further explore symmetry by creating cut-outs from folded paper.



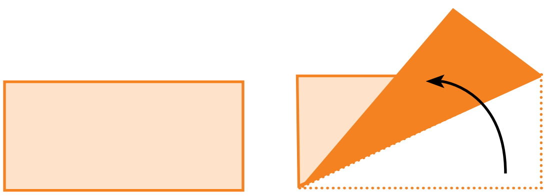



Folding and cutting has the advantage of encouraging pupils to recognise lines of symmetry by seeing the fold lines of the paper.

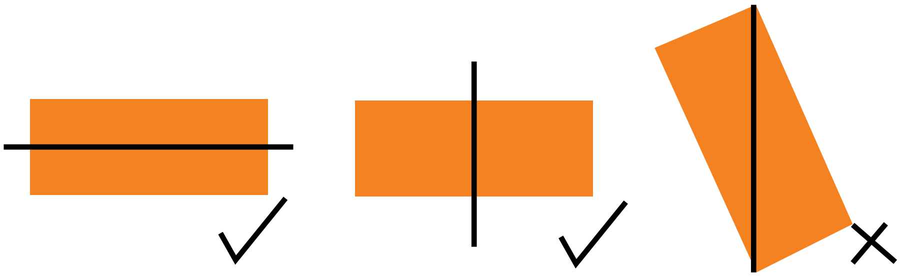
Without the use of lots of small mirrors or reflective silver cards, folding shapes will also help pupils to identify lines of symmetry on the various polygons that they study. So guide pupils to cut and fold a variety of triangles, quadrilaterals and other polygons. Many of the polygons have more than one line of symmetry - in a variety of different directions - so, as pupils handle, turn and fold the physical shapes, encourage them to find all the possible lines of symmetry for each shape. Ask each group of pupils to investigate how many lines of symmetry they can find for, at least, six different shapes.

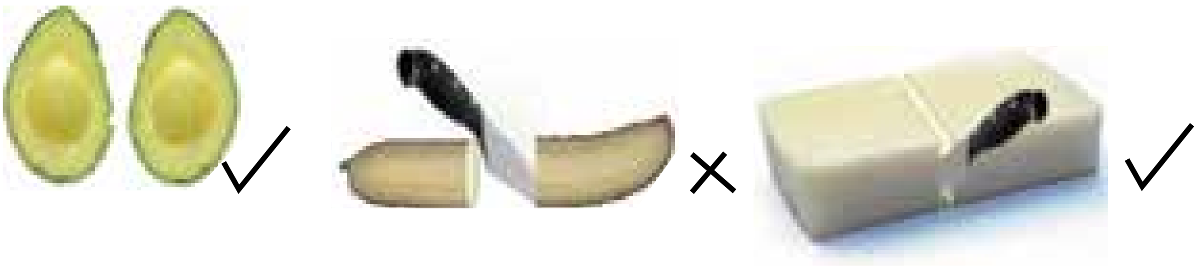
Pupils in Year 5 will add a description of symmetry to the properties of the quadrilaterals and other polygons that they study. They may be surprised to find that folding a rectangle in half along a diagonal does not produce a line of symmetry.



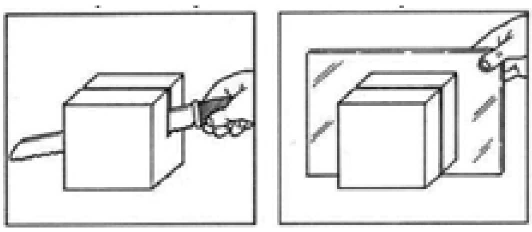
The rectangle does have two lines of symmetry but, for pupils to be convinced that these do not include the diagonals, they may need to see the rectangle turned so that the diagonal is vertical or horizontal: the lack of symmetry is then more obvious.



For large Year 6 classes, cutting solid shapes in half is not a practical possibility for pupils to do. But you can demonstrate 3-dimensional symmetry using some fruit and a knife …
… or, perhaps, a soft bar of soap.

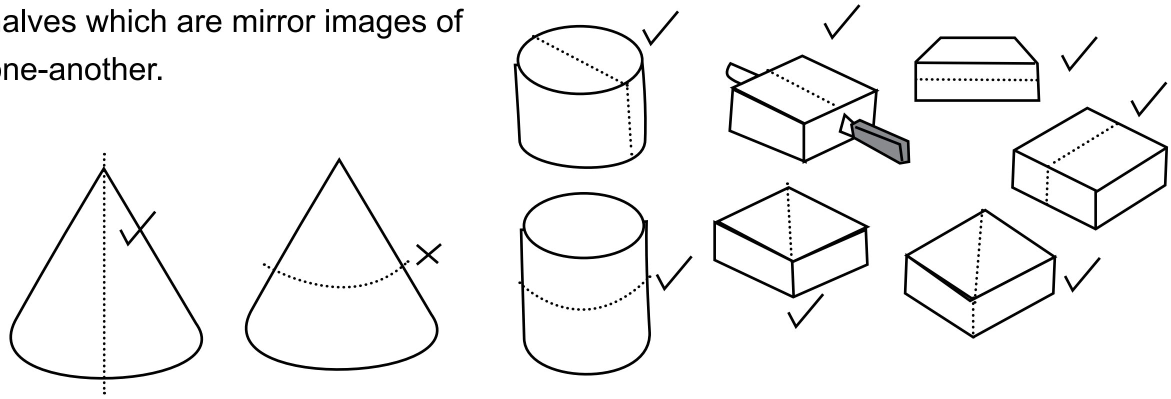


Once again the perception of symmetry is confirmed by the idea of reflection.



You will notice from this illustration that the symmetry is created, not by a single line, but by a cut through the solid (what mathematicians call a 2-dimensional plane of symmetry).

Pupils can investigate where a plane of symmetry can cut a solid into two equal halves which are mirror images of one-another.



Such an investigation would be difficult to do without some real objects to work with.

Think

What is the difference between a line of symmetry and a plane of symmetry?

Watch

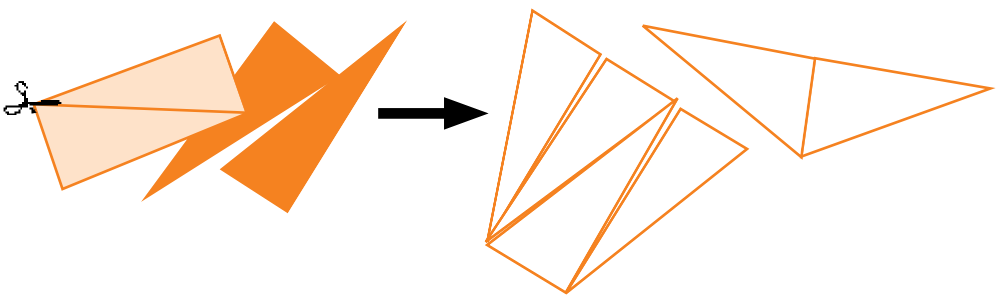
Watch the video clip MM11V1 on your phone. Notice that the pupils are able to draw very realistic pictures of some everyday solids to assist the teacher's presentation about symmetry.

Reflect

- Do you think that pupils may have been confused by the fact that they have drawn **lines of symmetry** on their 2-dimensional drawings of the 3-dimensional objects? Does this make the teaching of symmetry a difficult challenge for teachers?
- What materials will you use to effectively teach about …
  - … symmetry of 2-dimensional shapes?
  - … symmetry of 3-dimensional solids?

Work with your partner in school

Work with your partner in school. Cut two rectangles in half along a diagonal. Investigate how many different symmetrical shapes you can make by joining the triangles you have made edge-to-edge.

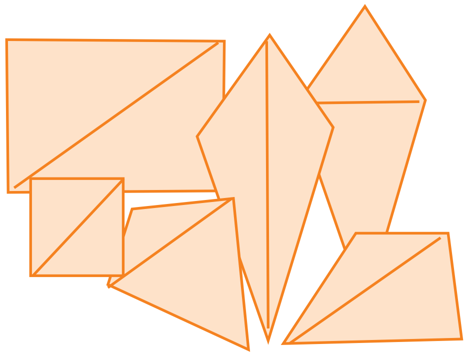


Plan a lesson for your class using this activity.

Think about how pupils can record their discoveries.

Making a poster? Sticking their shapes into their exercise books?

Summary of Module 11



The topic of Geometry is, essentially, about shape and how mathematics organises this study to recognise

- the properties that different shapes have, and
- the properties that different shapes share.

A key feature of this module has been the lines and angles which define the shapes. You know that the size of the angle at the intersection of two lines is measured by how many degrees of turn is needed to rotate from one line to the other. You will have noticed that

- the number of lines which create a polygon is the same number of angles that it has;
- every quadrilateral can be cut into two triangles by a diagonal line from opposite corners;
- triangles may be acute-angled, right-angled, or obtuse-angled; equilateral, isosceles or scalene;
- triangles and quadrilaterals can be isosceles or regular; … and symmetrical.

The vocabulary for geometry is extensive, with

- each shape having its own name to show its properties and the number of lines used to draw it;
- each type of angle having its own name depending upon its size compared with right-angles;
- lines being described as *vertical, horizontal, intersecting, perpendicular, parallel*, and so on.

There is a temptation for teachers to define all these aspects of geometry in a formal way and to expect pupils to remember the labels and definitions – especially when they do not have examples of the shapes to use in the classroom. However, pupils will acquire this knowledge and vocabulary more efficiently and more effectively from exploring the shapes and solids and from talking about them. Pupils need to engage in a practical way with real objects and with examples of shapes that they can cut out, hold, turn and fold. As you have realised, Geometry at primary school is dependent more upon a visual appreciation and less upon calculation. For their learning to be successful, pupils need objects to handle … and that means that you need to add a collection of shapes and some straight sticks to your mathematics teacher's toolbox. Collect these whenever you can.

The suggestions that follow emphasise the practical aspects of learning Geometry. While planning your lessons, don't forget that some children may have special learning needs; some may not have good motor skills or good visual skills and will benefit from working in groups and in pairs with other children who can guide and support them in handling shapes and objects.

Ideas to try in the classroom

When you try the following activities, you are encouraged to write a few notes about your experiences: what worked well; what the challenges were; what you did to overcome any challenges and whether the activities made a difference in your classroom. Being reflective about your own lessons helps you to become a first-class teacher.

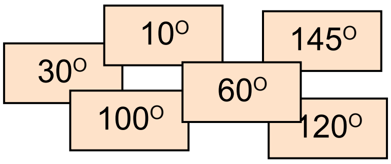
Try in the classroom 1

Topic: A sense of angle as rotation

Duration: 40 minutes

For **Step 1**, your lesson starter, ask pupils to make their forearms parallel.
"Turn one arm to make an angle of 45°."
"Now make an acute angle which is more than 45° but less than 90°."
"Show me a right-angle."

Step 2: "Now work in pairs. Face your partner and show her an angle of 20°.



Does she agree that your angle is 20°?" "She will now show you an angle of 150°.

Is she correct?"

Pupils take turns to challenge one-another.

For **Step 3** you will need at least five or six cards, each with different angles between 0° and 150° on both sides. Stand on one side of the room so that only one child in each pair can see you and the other child in the pair has her back to you. Show one card to the pupils facing you. The pupil with her back to you starts slowly rotating her arm from 0° to make an angle. The pupil who has seen your number card tells her partner to stop

rotating when she has reached that angle (which she has not seen and has not been told). The partner with her back to you must now guess the angle she has made. Is her guess the same as the angle shown to her partner? If her guess is within 10^{o} the girl facing you gets 1 point. If her guess is within 5^{o}, both pupils get 1 point.

Repeat the game with another angle card. After three or four games, stand on the opposite side of the room so that the pairs of pupils change roles.

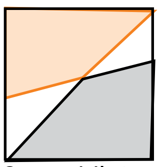
For **Step 4**, the final part of the lesson, pupils in their pairs each draw 10 angles, using a ruler, for their partner. Pupils exchange their drawings. They estimate the size of the angles drawn by their partner, writing their guess for each angle. Then they pass the work back to their partner who will check to see if he agrees or disagrees with his partner's estimates. To gain a point, each pair must agree on the size of each estimate and it must be a close estimate. The teacher should circulate during this exercise to ensure that pupils are making good estimates. (It will be good to have a protractor in hand to settle any disagreements and to give pupils a good visual prompt.) It will be important to remind pupils of sensible guidelines as you check their answers, for example: "This looks like half a right-angle, so do you think it is more or less than 45^{o}? Can you fold this square corner of paper to help you make a good judgement?"

Try in the classroom 2

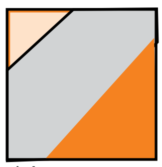
Topic: Recognising shapes

Duration: 30 minutes

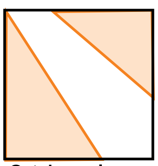
Draw 2 lines across a square, what different shapes can you find?



2 quadrilaterals
2 triangles



1 hexagon
2 triangles



2 triangles
1 pentagon

Try in the classroom 3

Topic: **Sorting shapes; lines of symmetry**

Duration: 40 minutes

Objectives: By the end of the lesson, pupils will be able to

- recognise characteristics of triangles or quadrilaterals
- identify lines of symmetry on 2-D shapes

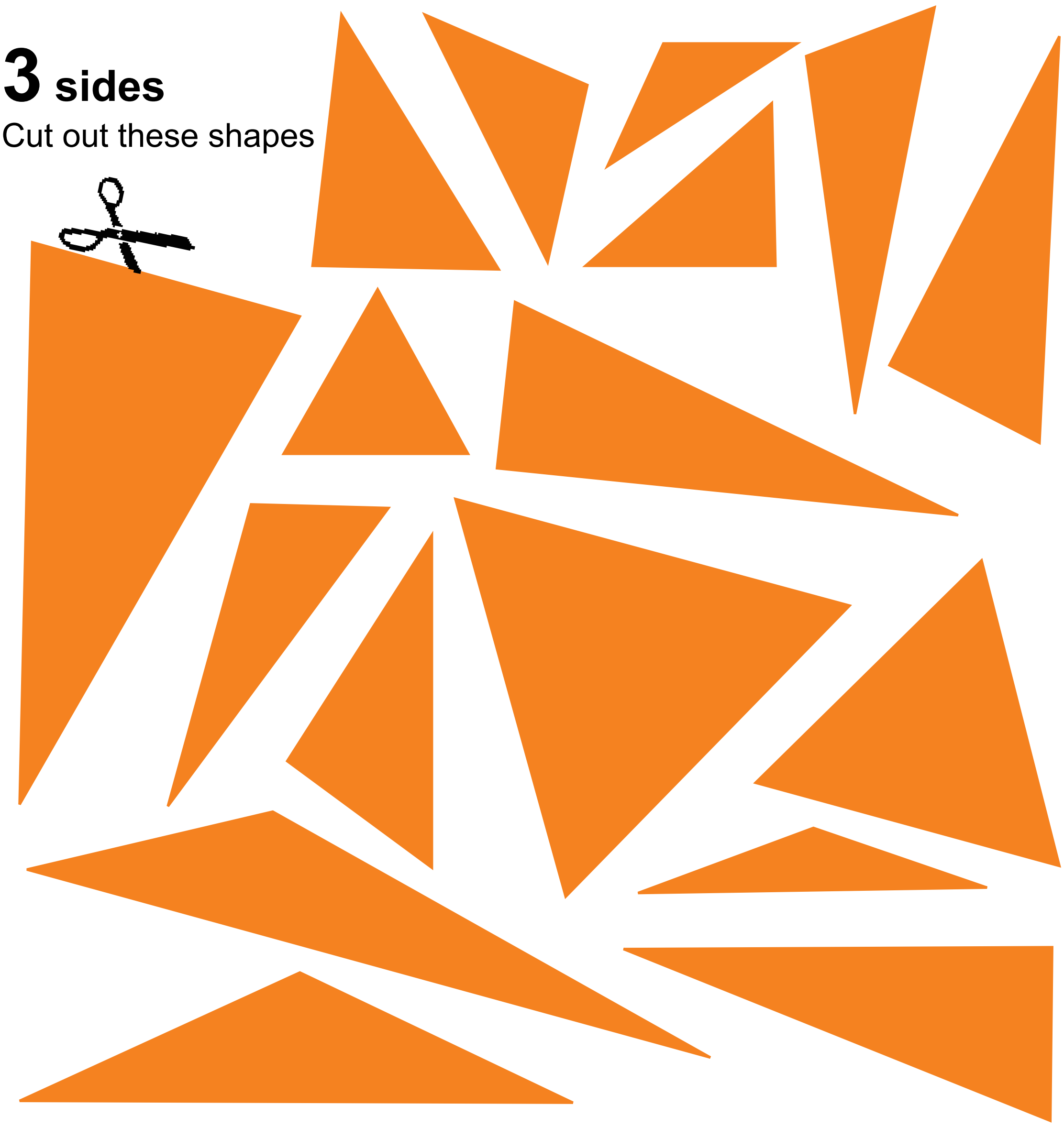
Teaching aids: scissors and cut-out sheet (thin card is best)

Previous knowledge: pupils know about the identification of plane shapes

Step 1 Pupils cut out the shapes on the sheet (Give pupils “**3 sides**” or “**4 sides**” or both according to your class.)

3 sides

Cut out these shapes



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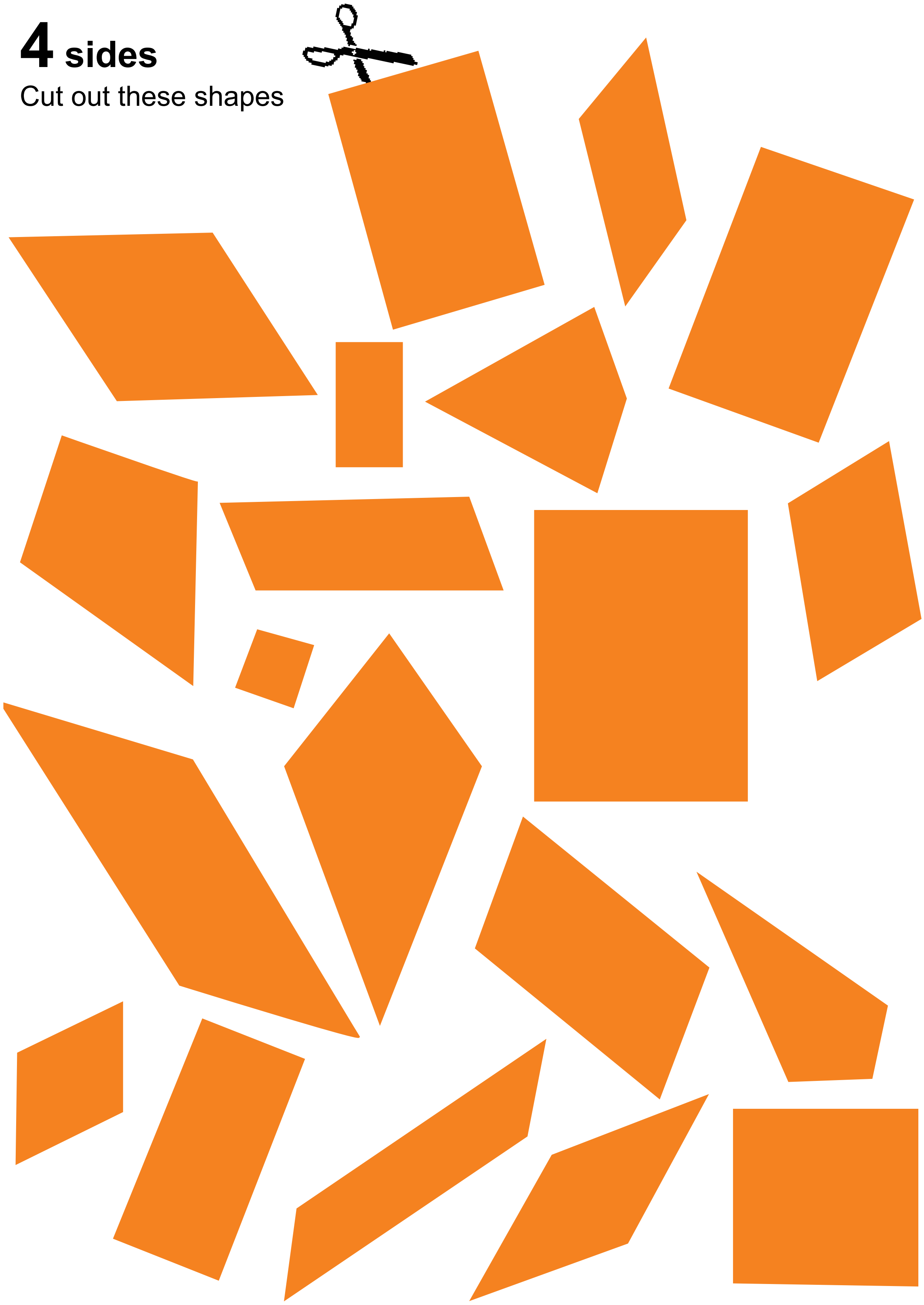
4 sides

Cut out these shapes



Step 2 Discuss the properties of one or two of the shapes so that common features can be identified.

Step 3 Ask pupils to sort the triangles into four groups of shapes (or the quadrilateral shapes into five groups). Tell pupils to name each group using the appropriate mathematical vocabulary.

Step 4 Discuss symmetry with the class and guide them to know that they can test for symmetry by folding a shape in half.

Step 5 Pupils investigate which of their groups have shapes which are symmetrical.

How many different lines of symmetry can they find for each of the shapes within the groups?

Step 6 Pupils are asked to categorise the groups of shapes by the number of lines of symmetry they each have.

Step 7 Pupil groups make their own notes to record what they have learned about the quadrilaterals and their symmetries. They could make a poster for display in the classroom. Teacher evaluates this work.

Experiencing change in your classroom

The suggestions for teaching geometric skills in this module rely substantially upon pupils participating actively in the lesson - not just by answering the teacher's questions but by making, cutting out and folding shapes or by using solids. It will be challenging for you to provide the suggested materials but you are encouraged to be resourceful to equip yourself so that you can become a more effective teacher. There is no doubt that pupils will enjoy lessons and learn successfully when they use these materials.

Looking to the future

Throughout this Teacher's Guide you have been encouraged to

- deepen your own understanding of the mathematics appropriate for primary school pupils;
- recognise a progression through the mathematics curriculum from Years 4 to 6;
- be aware of the underlying processes that enable pupils to learn mathematics.

Looking to the future, you are encouraged to be more adventurous in expecting children to discover mathematical ideas for themselves, with you preparing the ground for this and giving them guidance rather than giving them only rules to memorise. This will help pupils to

- have an ownership of the mathematical ideas;
- have a mastery of the mathematical skills; and
- be proficient in applying this knowledge and these skills.

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Persevere with collecting materials that will help you to teach mathematics in this more practical and efficient way. You have been recommended to develop a personal toolbox for teaching mathematics – mostly small things and pictures that you can use again and again to avoid the need to be always preparing new ideas. These no-cost or low-cost resources will enable you to re-use successful activities and to build your repertoire of good practice.

Be prepared to share your new experiences with your colleagues in school and be prepared to learn from them too. Take note of any good practice and use it to add to your own strategies for teaching. Both your Teacher Facilitator and your Head Teacher will be interested to know how successful you have been in making your lessons more engaging and enjoyable for the pupils.

Annex:

Sample answers for Modules 7 — 11

Module 7:

7.1 Counting in 10s and 100s

"**Watch**" answers

The teacher organised her pupils to make bundles of 100, so she quickly had 16 hundreds as well as about 20 more bundles of tens.

The Year 4 work on Numbers requires pupils to understand place value and to be able to count in thousands. The teacher used her bundles of 10 and 100 to build upon the Year 3 work, reminding pupils of the structure for counting up to 999.

The teacher in the video used 10 bundles of 100 short straws to show 1000. The children in the class used elastic bands to make the bundles. Because elastic bands deteriorate, it is often a good idea to use cotton or nylon thread to tie the bundles together.

"**Reflect**" answers

The teacher used her bundles of 100 to support the calculations using 100s. The children had to imagine bundles for calculations involving 1000s.

The bundles of 10s and 100s supported the transition from seeing the numbers to imagining the numbers. They helped to expand the pupils' conceptual understanding.

"**Work with your partner…**" answers

When the focus of the whole lesson is on mental work, it can require the teacher to be continually talking to the children and asking them questions, both to stimulate their thinking and to pose questions. This is never a good idea for more than a few minutes at a time because it makes the pupils passive. They cannot concentrate and absorb new ideas for lengthy periods of time. So it is a good idea to organise episodes of pair work or group work so that children can challenge one-another.

Pupils should also write down the calculations which they do mentally so that they have a record of the work done and so that their answers can be checked and, if necessary, corrected.

Pupils should also be able to write, in English words, the numbers which they use. For example to write the number 2107, both in figures and in words and know how the figures and words are different for the number 1207.

"Write the number which is four thousand less than four thousand and twenty."

"When Safiya counts down in 10s from 129, what is the first 2-digit number that she says?"

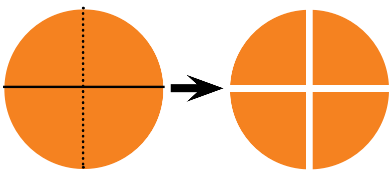
The use of a few banknotes is very useful for helping children to develop their mental imagery of counting in 100s and 1000s. The important idea is that the ₦100 note stands in place of a bundle of ten ₦10s; a ₦1000 is used in place of a bundle of ten ₦100s.

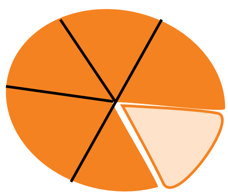
7.2 Fractions

"Watch" answers

Teachers often present an orange to the class to refer to fractions but, unfortunately, fruits are not very suitable - although we do often share a pawpaw (*gwanda*) or watermelon (kankana) - they are too messy to be cut to illustrate, for example, that 4 quarters are the same as 1 whole or that two eighths are the same as one quarter. A guava (*gwaiba*) is better but these are often not spheres and so may be difficult for illustrating the idea of equal parts – the essential idea of fractions in mathematics. This is why flags are a good teaching aid for fractions: pupils can fold small rectangles to create the fractions. They make good illustrations. They often require pupils to add internal lines to create the equal sections (which is a good challenge) and they make learning interesting.

A successful aid for introducing fractions is a selection of paper or plastic plates (you can use them first for food at home before cutting them into halves and quarters). These offer the possibility of combining fractions as well as recognising how many parts make a whole.





In the second illustration the plate has been cut into sixths and shows that

$1 - {}^{1}/_{6} = {}^{5}/_{6}$

7.3 Decimal Fractions

"Watch" answers

The teacher gave each group some decimal numbers to add together. She asked one member from each group to write their completed calculation on the chalkboard.

"Reflect" answers

This group of boys need to recognise that 16 tenths make a 1 whole and 6 tenths. They should write these two parts of their answer in the correct columns for units and tenths.

The mistakes made by the group of boys emphasises the need to ensure that pupils are familiar with the idea that 10 tenths make a whole 1. So you could suggest to the P4 teacher that pupils do these three exercises illustrated in the summary of Module 7:

"Work with your partner…" answers

1. The method illustrated in the video only shows the positive organisational aspects of giving group work and of using pupils to demonstrate a correct method. The teaching does not offer any explanations about the decimal fraction format and it doesn't help pupils to understand that decimals are closely linked to measurements. Activities such as those illustrated in the summary (reproduced above) will help to overcome the challenges in teaching decimals so that pupils will know, for example , that 0.8 + 0.2 = 1.0 or, simply, 1 because these decimal fractions mean ${}^{8}/_{10} + {}^{2}/_{10}$

2. It would be tedious to expect children to colour in a 100-square to show that one line of 10 squares is one tenth of the square, although this would be a good class demonstration. You divide both numerator and denominator by 10 but Year 4 pupils will not be used to cancelling factors in a fraction (which is part of

Year 6 learning). Perhaps the simplest idea is to show that the denominator is 10 times bigger that the numerator.

3. It is important for pupils to know that decimal fractions are simply a different way of writing the fractions that they already know.

7.4 Ratio

"Think" answers

1. You may have found the most common mistake that pupils make with ratio is to describe the difference between two numbers rather than to compare their sizes in a multiplicative way. They might say, wrongly, that the ratio of fingers to hands is "8 more" rather than "five times" when comparing the number of their 10 fingers to the number of their 2 hands.

2. An important aid for learning about ratio is the notion that "for every hand, there are 5 fingers. The idea of comparing the number of fingers to hands or sweets to children is hard to convey on a chalkboard. Children showing their hands, or sharing 20 sweets for 4 pupils, helps to build an understanding of "*for every ☐… there are ☐…*"

"Watch" answers

The teacher's introduction concerning **sharing** may conflict with the idea of fractions where it is important that sharing is done equally. The children's examples of sharing items with a friend and the teacher's example of sharing an orange suggested that are thinking of sharing things equally.

Ratio is more likely to be used when the sharing is not equal. For example N1000 could be shared unequally between two people so that one person has N800 and the other person receives ₦200. The money has been divided between the two people in the ratio 4:1; it has not been shared equally which is our normal concept of sharing. It would not be wrong to suggest that ratio is concerned with **unequal sharing** but that would exclude comparisons of quantities in the ratio 1:1 Ratio is more often a means of **comparison**, not of sharing.

The teacher's example of comparing the number of pencils and the number of books is a better introduction because it shows that comparisons can be made between numbers of different objects. His materials (bottle tops, stones seeds and straws) confirmed this idea.

By checking each group's ratios of different quantities, the teacher was able to confirm that each group had made appropriate comparisons. However, when pupils had compared 4 seeds with 2 bottle tops, he did not encourage them to recognise that for every bottle top there were 2 seeds - so pupils did not understand that the ratio of seeds to bottle tops was 2:1. Similarly the pupils who demonstrated 6 straws and 4 stones were not encouraged to recognise this was a ratio of 3:2 Hence, it is difficult to witness any real learning. The children did not need to use any new knowledge to count the number of books, pencils, seeds or bottle-tops that the teacher used as his teaching aids. So, what was the point of the lesson?

"**Reflect**" answers

1. ● Slow learners simply need more time to engage with the learning activities. They benefit from learning tasks being broken down into small steps. They also benefit from repeating an activity in similar but different ways: for example, comparing 6 books and 4 pens; comparing 6 stones with 4 nuts; comparing 6 oranges with 4 guavas and recording each comparison before recoding that these objects are all in the same ratio 6:4

   Dividing each group to have lots of sets containing 3 objects and 2 objects will allow them to recognise that whenever there are 6 of one and 4 of the other, these can be rearranged to show that for every 3 of one type there are 2 of the other type.

   ● For visual learners, the use of real objects will allow them to understand the concept of ratio. They will be able to see and to judge how many times larger one group is than the other. The suggested necklace activity will be particularly useful for visual learners.

   ● A child with hearing difficulties will depend more upon reading about and seeing the groups of objects. Therefore, demonstrations which physically compare two groups of real items and the recording of the comparisons on the chalkboard will benefit their learning.

2. The teacher could split each group of 4 pencils and 6 books into two groups. each group would have 2 pencils and 3 books. This would demonstrate that 4:6 can be simplified to 2:3

7.5 Percentages

"Think" answers

1. If 40% are girls, the boys must be 60% because one whole is 100%.

2. If petrol was N160 per litre before the increase, a 25% increase would add N40 to the price, making a new price of ₦200. If the new price is to return to N160, then there now needs to be a reduction of 20%

"**Watch**" answers

The teacher provided encouragement by providing a 100-square for shading in percentages which totalled 100%. This is easy to provide but only if you have access to a copier.

He also provided some items on which he had put price labels. Pupils were able to calculate their discounted prices, making the activity realistic. This is an easy task, taking only a few minutes preparation.

Pupils always respond positively to praise. But make sure that they have done something to earn the praise rather than just say "*Clap for yourselves*" if they have only repeated something that required no effort or no learning.

Because a 10% discount is easy to calculate, the teacher could organise a quick quiz of five questions in which pupils have to write down the new price of five items. Pairs of pupils would have to agree on the new price before the teacher requests them to give an answer.

Another quiz could ask pupils to match 5 pairs of prices, showing the original price and the sale price. An example was given in the Summary section.

"**Work with your partner**" answers

The episodes in this lesson showed that pupils already know how to calculate simple percentages. But the area not treated was how to express one quantity as a percentage of another: for example, "Only 72 women were among the 360 doctors who qualified. What percentage of the doctors were women?"

The Section on Percentages suggests a sequence or progression through percentages that will take at least five lessons to teach.

Module 8:

8.1 Addition and Subtraction

"**Think**" answers

- All statements are **True**

"**Watch**" answers

The teacher is very articulate, she demonstrates each step, makes sure that pupils understand what she is doing at each stage.

"**Reflect**" answers:

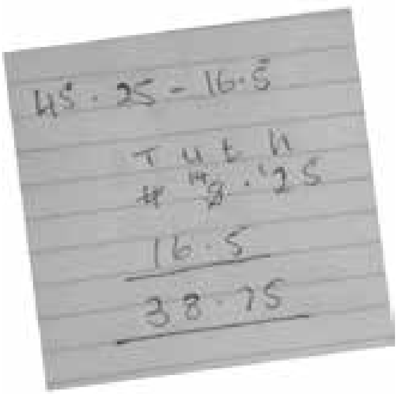
To organise the group work

- the teacher set questions on individual cards.
- she told pupils to work together and agree on the answers, making sure that the group leader checks that everyone in the group understands what the leader has written.

The pupil whose work is shown has not carried over the tens into the next column on the left.

He will need to:

- write headings above each column;
- line up decimal points and each column of digits;
- carry the *1*s into the next columns on the left.



What errors have been made by this pupil?
How will you help her to recognise the errors?

“**Work with your partner**” comments:

The example of an expanded addition may be very helpful for some children. It may also be a distraction for others who want to learn a straightforward method without having to modify their previous method of working.

Some children will be reassured to know how the partial sums appear in the columns because the written method records their thinking.

Only you will be able to judge which method is best for the children in your class and when to move forward to a more sophisticated written method. Don’t be afraid of trying several different strategies until you find what works best for the individual pupils in your group.

When learning how to subtract with the column method, it is very common for pupils to write the difference between the two numbers in a column even though the top digit is smaller than the bottom digit.

This mistake emphasises that pupils need to develop understanding of what they are doing – not just follow a half-remembered rule.

It is best to be direct and ask “Why can you not take 9 away from 7? What can you do to allow 9 to be subtracted?”

Often, providing a correct answer such as shown in this picture,and asking “*Why has Nafisa written “8” under the units and not “2”* ? is sufficient for a pupil to recognise his own error.

8.2 Multiplication

“**Think**” answers

- Most people use both interpretations of the multiplication at different times.

  This may mean that your pupils may be unsure of whether 23 x 5 means 23 lots of 5 or 5 lots of 23. *Does this matter?* No, as long as they know what they are doing.

- The grid method enables pupils to “see” the separate multiplication products within a calculation. This helps them to articulate the steps of the column method and so it is a good bridge to support the progression.

- Teaching “x 10” as a 2-digit multiplication involves multiplying by zero from the Units column and then multiplying by 1 from the Tens column. This would be confusing! Encourage pupils to treat 10 as a single digit and multiply by 10 as a single operation. Treat “x 11” as a single mental operation too !

“**Watch**” answers

You may have liked the teacher

- being very clear and emphatic about the four smaller steps that all contribute to the long multiplication answers;
- asking pupils whether they preferred to multiply using the four steps or the condensed method version which relies more upon their mental skills;
- stepping back to allow the two pupils to explain their groups’ answers.

  You may have wanted to improve the presentation by:

- always writing the individual multiplications in the same consistent order so that pupils won’t mix their routines;
- setting more than just two questions to each group;
- asking some easier and some harder questions so each pupil has an appropriate challenge.

“**Reflect**” answers:

Advising Hamisu and Nafisa:

- Hamidu explained the step-by-step procedure for 34 x 21, starting with multiplying 34 by 1. Why would he think that this was a necessary two-step multiplication that needs writing out in full detail? He should recognise that 34 x 1 = 34

- Nafisa showed how to multiply the first part by demonstrating how to calculate 25 X 3 as a two-step multiplication. Her obvious good ability suggests that she could have done this in her head … 2 x 25 = 50; 3 x 25 = 75,… And having got the answer 75, she goes through another two-step routine to multiply 25 by 30. Does she not recognise that this will simply be ten times bigger than her answer to the first part? As 25 x 3 = 75, then 25 x 30 = 750.

The children's explanations show that they are blindly following the routine of four unnecessary steps when they are both clearly able to think through the multiplications with a much higher degree of mental ability. Their demonstrations suggest that Hamisu and Nafisa are being discouraged from having good functional skills because they are being asked to blindly follow a routine. There may be some children in the class who will need to rely upon such a supportive routine but Nafisa is well-able to carry out the long multiplication without it.

Do you teach your pupils to think? Or do you only teach them to follow routines?

Starting with xT or xU:

It doesn't make any difference to the final answer if you multiply the quantity by Tens and then the Units or if you multiply by the Units first. However, as a teacher, you should always be consistent, using the same order when you teach long multiplication. Later, when pupils are confident with the procedure, they can discover that the order for multiplication is unimportant.

8.3 Division

"**Think**" answers

Pupils have different ways of thinking through division calculations. These are likely to be a mixture of grouping, subtracting and inverse multiplying. Encourage children to talk about their strategies for dividing 27 ÷ 3

"**Watch**" and "**Reflect**" answers:

The different pupils will have different levels of confidence when choosing how big a chunk to subtract. Their confidence will depend upon their ability to multiply mentally.

The teacher encouraged her pupils to choose a subtraction that they would feel comfortable with. Sometimes this might mean that some children will choose smaller steps, making the long division longer, but it is the process which she is teaching.

She was encouraging each pupil to have ownership of the process.

Pupils will choose more efficient and bigger subtraction chunks when they feel confident to do so.

For pupils to have functional mastery of these mathematical skills, they need to have full understanding of why it appears that the decimal point moves. Knowing that ÷100 gives the same result as (÷10, ÷10) enables pupils to confirm their wider understanding of the mathematics underpinning the mathematical ideas.

Many of the pupils in this class were already able to divide by 10 and so more-challenging questions could have been given to most pupils.
For example, 2.67 ÷ 100 and 0.267 ÷ 100.

8.4 Estimation

"**Reflect**" answers:

Do you think that the second part of the lesson will have asked pupils

- *to do another exercise in the same way ?* No, the mental process has been made clear.
- *to do group work activities in the same way ?* No, it's not necessary. The teacher recognised that the pupils were able to use near rounded numbers to make good estimates.
- *to do another exercise of calculations where pupils only write an estimated answer ?* Yes, that would be appropriate. In the first part of the lesson the pupils showed that they knew that their estimated answers gave close approximations to the actual answers.
- *to do group work where pupils only write estimated answers for the questions given and then swap answers with other groups to decide whether their estimates are sensible and reasonable.* Yes, that would be good.

The lesson which you saw continued with two more parts, with pupils only making estimations.

In the second part of the lesson, the teacher asked pupils only to give estimations for the answers to five different calculations which she wrote on the board. The groups of pupils had to agree on sensible estimates for each of the five calculations. Then each group leader reported to the whole class how they had made one particular estimate. The teacher led the discussion so that pupils agreed about why the answers were appropriate.

In the final part of the lesson, pupils had to agree on sensible estimations for the answers to five more calculations. When each group had finished making their estimates for these calculations, they passed their list of estimated answers to the next group who then agreed or disagreed with their estimates.

The whole lesson focused upon building the pupils' confidence to be able to make their own mental estimates for calculations that would not need to be done accurately.

"**Work with your partner**" answers

If pupils are going to make an accurate calculation, the only purpose of making an estimation is for them to have some idea of the calculation they are going to do and to have a rough of idea of the answer they expect. This will enable them to recognise if an answer is not sensible or if they have made an error in the calculation. This estimation should become a mental skill.

If an accurate answer is not needed, pupils make an estimation because they only need to have a rough idea of the amount. Plan a lesson to include the group work similar to that described in the "Reflect" answers.

8.5 Indices

"**Think**" answers

The indices in the multiplication 10 x 103 are 1 and 3. There is no need to write in the power of the first 10 because it is obvious that the first number is only one ten.

"**Reflect**" answers

Using the prime factorisation was an excellent introduction because the teacher used a skill which the pupils already had to introduce a new idea of the mathematical shorthand of indices.

8.6 Using Brackets / Order of Operations

"**Think**" answers

- True. If you don't know the order, the answer you give may be different from that intended.
- True. Using brackets avoids confusion because they tell you clearly what to do first.
- False ...and neither rule is more important than correctly using brackets.

"**Reflect**" answers:

The children's excitement was very visible. Even when some pupils missed getting the ball in the box, there was evident pleasure in trying to score. Such a simple game to organise!

It focused the children on a reason for doing the mathematics. The teacher's method of recording the game's results enabled him to make use of brackets.

Module 9:

9.1 Tally Tables

“**Think**” answers

1. True. Making a Tally Table is a dynamic way of collecting data as it happens.
2. True. Tally marks record individual items one by one.
3. No. Tallies are grouped in fives. Humans can subitize a group of 5 but not a group of 9 or 10. It would be harder to recognise when a group of tallies reaches 9 and then to make the cross tally the 10th one. So tallying is done in groups of 5.
4. No. The fifth tally completes the group of 5.

“**Reflect**” answers

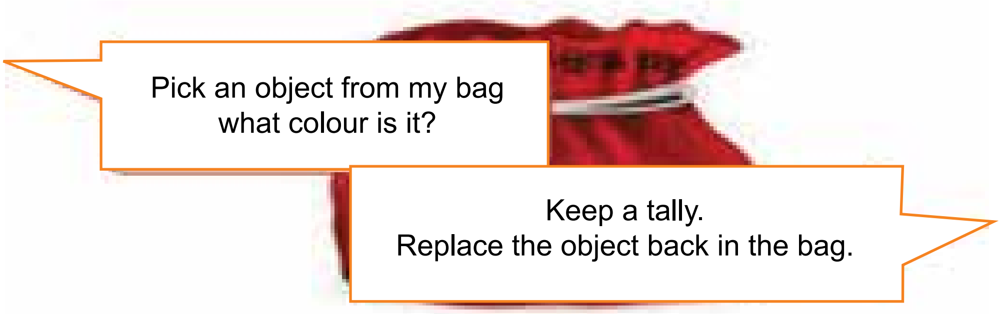
The surveys of cars passing the school gate provide the real experience of making a tally table. The information can only be obtained by watching the vehicles pass and recording them one-by-one. You can only know the answer to questions like

> *“How many vehicles pass along the road every hour?”* or
> *“What are the most popular car colours?*

by such surveys.

If you are able to carry out a vehicle survey, the amount of time needed will depend upon how busy the road is. For a busy road, 10 minutes might be enough to record about 30 vehicles; a quieter road may need 20 minutes but both of these will allow the children to make a real survey. As the teacher, you will need to make sure that the pupils have paper and pen for recording the data in a table. You will also need to ensure that they stay safely inside the school compound.

Although the favourite fruit survey is suggested to give pupils an experience of making a tally, not everyone on the class will actually do the tallying. And, as you have been previously warned, the same information could be obtained without doing a Tally Table.



Picking a coloured stone or sweet from the teacher's bag provides a good Statistics exercise, particularly if everyone makes a tally table as the stones are selected. Copying the teacher's table doesn't provide pupils with the real experience of creating a tally chart. But this experiment does have the advantage that you can use the results to ask pupils to think about what the Tally Table shows to help predict the actual number of colours in the bag.

"**Work with your partner**" answers

You may like to try this experiment with the pupils in your class. You will need at least one pair of books for each group of pupils.

"*Do children's books have shorter words than adult books?"* A survey by your class can suggest that this is true (or maybe untrue, depending on what you find) but to answer the question definitively you would need to take a much larger sample to survey. If this was true for the comparison of, say, 100 books, you would be entitled to give a definite answer.

9.2 Pictograms

"**Think**" answers

Pictograms give immediate visual information.

Because the number of objects which each pictogram represents is given in the key, there is no need for a scale or number axis at the side of the picture.

A pictogram can be chosen so that the symbol is linked to the data being displayed and so adds an interesting feature.

"**Reflect**" answers

1. The Tally Table and Pictograph both
   - categorise data to enable it to be read;
   - show the totals of each category in the chart;
   - make it easy to see which category is chosen most frequently;
   - make it easy to compare items of data.

The Tally Table is a recording device to store data as it happens.
The Pictograph is a displaying device to make data easily "readable" visually.

2. The answer to question 1 suggests that Ibrahim should
   - have used a Tally Table to collect his data, asking each person one-at-a-time about which days they go to the mosque; and
   - use a pictograph to display the information.

"**Work with your partner**" answers

1. The "Sales of Kosai" pictograph will be of interest to the children because some of them may sell kosai after school or at the weekend. (… and most children enjoy eating them.) There are lots of questions to ask:
   - how many kosai are represented by the half pictogram?
   - how many kosai are sold on each day?
   - which day has the best sales?
   - how many more are sold on Tuesday compared to Monday?

The pictogram can be the feature of an interesting lesson in which pupils learn how to make their own chart. Because the kosai pictogram represents 10 kosai, you will first need to teach the class how to use a pictogram to represent 1 item before moving on to this next stage. A good first lesson would be for the children to draw a group pictograph to show the number of their brothers and sisters.

A second lesson could be for them to draw a pictograph showing the results of their Tally Table which recorded their favourite fruit. Because there are many children in the class, you can discuss with them how to use a fruit symbol to represent 5 people to avoid them having to draw one pictogram for each pupil. This discussion will give them an insight into why symbols are used.

2. You would reject the pictograph of cars because
   - The pictograms are not all the same size.
   - If the car in the key is a different size to those in the pictograph, are the smaller cars meant to indicate less than 5 – you couldn't know!
   - The cars are not lined up starting on the left so it makes a visual comparison difficult.
   - The chart has no title or suggestion of what information is being shown.

3. The pencil chart is like a pictograph because it uses symbols to show the data. But these symbols are not repeated to show the numbers. It's not like a pictograph because the pencils are not all the same size and so numbers have had to be added to give the missing information.

9.3 Bar Graphs

“**Think**” answers

You should satisfy yourself that you can answer all the questions for pupils that are suggested in orange in Section 3.

The Year 5 questions which involve some deduction are important – such as the question about owls’ food: “*Do you think that the owls got more food from insects or from small birds?”*

Pupils should be encouraged to be perceptive about the information. Pupils will recognise that more insects are eaten than frogs, but they should reason that one small bird or frog will probably provide as much nutrition as very many insects - so the data does not indicate the importance of items of food for owls.

“**Practice**” question

Because the numbers are small, a scale in units up to 10 will be appropriate for your graph.

“**Watch**” comment

The teacher succeeded in engaging the pupils because he used the pupils’ favourite colours to create the graph. You may question why he changed the number of pupils choosing RED from 15 to 14 when the graph axis, going up in two’s, could have allowed 15 to be recorded halfway between 14 and 16.

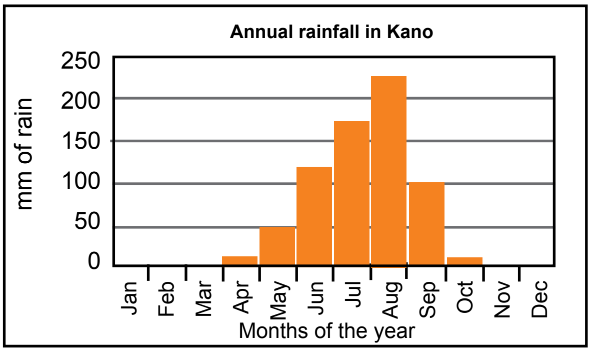
When pupils drew their own graphs, they needed pencils and paper.

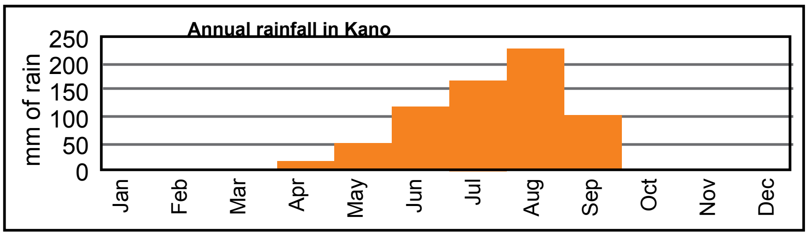
“**Reflect**” answers

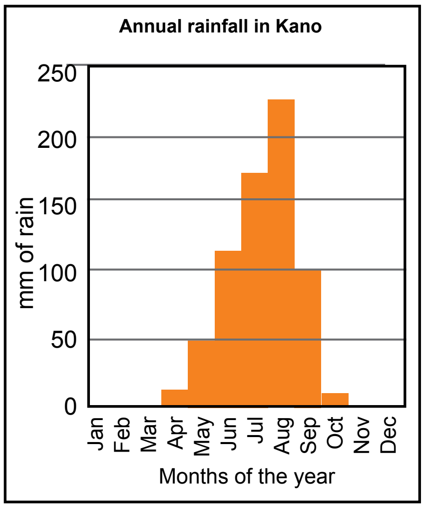
The children managed very well. They used rulers and pencils to enable them to draw suitable graphs.

Fatima’s explanation was very clear but her group could have efficiently used a scale of 20s on the frequency axis if they had thought about the numbers in the data.

How do the scales on these graphs give a different idea of how much rain falls in Kano in August?







9.4 Mean Average

"Think" answers

When Hassan said the average height of his goats was 55cm, he was just thinking about the majority of goats. Mohammed's calculation of all the goats included the baby ones too, so the mean average was lower.

When 20 mangoes are shared equally by 4 people, they each get 5 mangoes. It's a definite fixed average calculated by sharing. When Hassan's goats vary from one month to the next, between 54 and 66, the average is 60. But if the number of goats in two months varies between 56 and 70, the average for these two months would be 63. Hassan's notion of a herd of 60 goats is a different sort of average. Over time, the number of goats oscillates either side of 60. Sometimes more, sometimes less. It's just one number that varies. You would need to consider the number of goats over several months to calculate the mean average. For example:

January	February	March	April	May	June
60 goats	54 goats	66 goats	70 goats	56 goats	54 goats

“**Watch**” comment

In fact, the teacher did not explain the meaning of “mean”. He only explained how to calculate it. Pupils need more than a knowledge of rote procedures to be able to have an ownership of the mathematical ideas. *Would you have explained **why** the average of 7, 13 and 10 is 10?*

“**Reflect**” answers

- The teacher used a practical example, giving the pupils 3 different amounts of straws and asking them to share the straws equally. This clearly demonstrated the idea of grouping all the numbers together and then dividing by 3. The pupils did the sharing physically and so their practical experience made writing on the board unnecessary.
- All the pupils’ examples were of adding three numbers and dividing by 3 to find their mean. The teacher avoided any problems by ensuring that all the totals came to multiples of 3 and so this was rather a contrived exercise. Finding the average of a different group of numbers, such as 20, 23, 26 and 29, requires pupils to divide the total (98) by 4. This, and other variations, will ensure that pupils will know to divide the total by the number of numbers, not always by 3. This calculation for the mean of four numbers gives an average of 24½ (or 24.5). Some pupils will find the decimal fraction answer for a mean average more puzzling to understand. You will have noticed that, by giving the pupils three single digit numbers to find a mean average, the teacher made this a very easy exercise in order to focus on the strategy. However, with such simple calculations, he should have expected pupils to answer this exercise mentally.
- These pupils may not have a perception of the “mean” as an average for a group of numbers because they only learnt the formal rote procedure of adding three numbers and dividing by 3. Their answers were just defined as “the mean”. The notion of “average” was only implied by the equal sharing of straws and the word “average” was never used (perhaps because it is a difficult idea to pin down or explain).

“**Work with your partner**” answers

A lesson using small bags of groundnuts is a very successful way to help pupils have a conceptual understanding of an average amount. It builds the notion that there is a “central” number which gives an indication of a representative value for a group. This is a difficult concept to teach because it is difficult to define an average in a single comprehensive way.

You can understand this difficulty by considering the answers to questions such as
"What do you do on an average day?"
"What is the average speed of the bus?"
"What is an average ability?."
"What is an average shoe size?"
"What do you eat for an average meal?"
"What's the average size of Hassan's goat herd?"

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9.5 Median, Mode and Range

"Think" answers

Check with your partner in school that you have matched the methods correctly with the types of average.

"Watch" comments

The teacher uses a sample of nine pupils' weights to demonstrate to the class how to find the Median and Mode averages for this group.

"Reflect" answers

If the learning objective was only for pupils to know the procedures for finding the Median and the Mode, then the lesson objectives were achieved. However, the teacher did not mention the word "average", nor was there any discussion about why these calculations were being done. The pupils simply followed a procedure without any understanding of

- why Medians and Modes are calculated;
- why the median and mode in all of the examples were the same number;
- why the data should be arranged from lowest to highest;
- whether it was important that the calculations were always a group of nine numbers;
- how to find the median of an even number of items;
- how to find the mode of non-numerical data, such as the colour of cars passing the school;

- how to find the mode when two or more items have the same frequency;
- what it means if the Median and Mode are very different numerical amounts;
- comparing a range of, say, 8 in a sample of items ranging from 0 to 100 and in a sample of items ranging from 0 to 10.

It is never satisfactory for pupils to remember a rote procedure without having any understanding of what they are doing. So, in the wider context of pupil learning, the objectives concerned with pupil ownership and mastery were not achieved.

In the examples discussed in this section of the module, the median and mode were all different quantities. Understanding when and why these might be the same for a set of data will be of advantage for pupils.

"**Work with your partner**" comments

- To find the middle item in a sample of data it is much more efficient if the data is ordered from lowest to highest. This also makes it easier to recognise the two central items when there is an even number of items. The median for a set of even items is always the midpoint between the two central items.

- Finding the averages for the class' heights, weights, ages, etc makes the lesson more meaningful for the pupils.

- Pictures such as the Sumo wrestlers also makes the work on averages more interesting. So keep any interesting pictures in your collection of teacher tools. When you do not have any such pictures, pupils can collect data on items that can easily be found - such as the average length of a yam; the average size of a ripe mango; the average length of a plantain finger.

Module 10:

10.1 Length

“**Think**” answers

Pupils should gain sufficient experience of measuring and calculating with metres and centimetres so that they know roughly how long these units are. A centimetre is about a finger width; a metre is a large pace.

“**Watch**” answers

The teacher asked children to think of contexts where length is important; the pupils thought about measuring cloth for a dress dress but the teacher didn’t describe the notion of length as being the distance between two points or between the two ends of the cloth. However, he demonstrated the idea by asking pupils to estimate the length from “here to here” across the chalkboard.

The pupils first of all used their forearms to estimate, then measure, lengths. In the group activity, the pupils used tape-measures to measure more accurately. Pupil participation was guaranteed by asking them to cut rectangles to specified sizes — a good idea to engage pupils because the request required them to measure accurately before cutting the shapes.

“Reflect” answers

1. Your pupils will probably like Lesson B because they will enjoy measuring one-another. Lesson A will sound like more work because of the writing they will be required to do but taking turns to measure the objects can also be appealing if there are interesting objects to measure.

2. Providing a tape measure for every pair will be a challenge for Lesson B. This is basically a matter of cost for the school unless you can bring lots of tapes. This is not very realistic.. In Lesson A it will be demanding to organise the group work and to ensure that everyone takes part in the measuring of the objects; you would have to think about what pupils can do while they are waiting for their turn to come to the central table. Two or three tables with the same apparatus on them will be better. An alternative would be to have just two objects on a lot of tables and pupils rotate around the circuit of tables with the teacher organising the times for changing to the next table.

3. Both lessons can be very interesting for pupils: practical experience of measuring, allowing movement in an organised way, measurements provide for good mathematical learning; tasks make use of the measurements to learn about comparisons using difference and ratio.

4. Each lesson has a different focus and offers different potentials for developing mathematical thinking. So it would be beneficial to try both lessons.

"**Work with your partner**" comments

The objects listed are everyday things which are easy to measure. Can you add any unusual items to stimulate pupils' interest: a bicycle (wheels, handle bars, seat, …); an empty box with smaller empty boxes inside; a pineapple; …

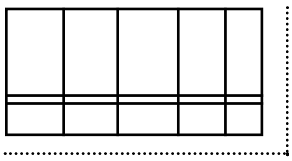
10.2 Area

"**Think**" answers

1. For the area of this trapezium you would need to know the length at the top of the rectangle (same as the top of the trapezium) and the width of the rectangle (height of the trapezium) to find the rectangle's area. You would also need to know the base of the triangle and the height of the triangle: it's area is half of the rectangle defined by those sides. Because the base of the triangle is the difference between the trapezium's base and top, you only need to know that base and top and the distance between them.

2.

$4^1/_2 \times 2^1/_2 = 11\ ^1/_4$



The diagram makes it obvious where the ¼ comes from because you can see that the halves can pair up to make three whole centimetre squares. The half of the half produces the quarter. A visual explanation helps pupils to have confidence in the procedure they are learning.

Such a diagram also reinforces the notion that multiplying length x width is a way of calculating how many centimetre squares are inside the rectangle.

"**Watch**" comment

The teacher asked pupils to measure a variety of different rectangles which she had brought to the class. Pupils were eager to participate because they had practical work to do to generate the lengths needed for the area calculations.

"**Reflect**" answers

Sadly, the teacher's demonstration only referred to the rule of multiplying length x breadth. It is likely that pupils cannot say why the area is 18cm^2 unless identifying this relationship had been a feature of a previous lesson.

"**Work with your partner**" comment

The area of each of the shapes illustrated can be found by adding three or four rectangular sections together. The areas can also be found efficiently by subtracting the hole from the area of an imagined surrounding rectangle.

10.3 Volume and capacity

"**Think**" answers

The use of the word "capacity" in a phrase like "the capacity to succeed" derives from its mathematical meaning of "the amount that can be contained". The meanings are very similar.

"**Watch**" comment

The teacher provided resources for children to acquire a concept of capacity and the first part of the lesson focussed on the capacity of containers, measured in litres. Pupils were eagerly attentive to Asibi as she measured the capacity of a jerry-can.

The pupils' activity which followed asked pupils to estimate, then measure the volumes of various amounts of water in several bottles. The activity was stimulating and largely successful: the practical aspects helped pupils to have a mental picture of the size of a litre, half litre, 300ml, etc. as well as providing the opportunity to read a scale.

To draw a distinction between capacity and volume, the pupils were later asked questions such as "*You said the 2-litre bottle contained a volume of 600 millilitres of water. What volume of water could be added to fill the bottle to capacity?*"

10.4 Weight

"Think" answers

The most important skill for reading a scale is to recognise the gap between measurements given so that the value of each division can be known.

On this scale there are four intervals between 100 and 120.
So each division must be 5
The reading is 115

On this scale there are five intervals between 40 and 50.
So each division must be 2
The reading is 42

"Watch" comments

The teacher provided several weight labels to each group so that they could calculate the weight of all the items in the group's imagined shopping basket.

Sadly, the video clip only showed numbers being manipulated. So there was no actual experience of weight for the pupils. Nor any experience of the key skill of reading a scale.

"Reflect" comments

Perhaps the teacher could have brought real items rather than labels. In the light of the previous comments, a Head Teacher would recognise that giving a definition of weight does not provide any experience of what a kilogram weight feels like. As the teacher mentioned 600g of sugar, this and other items could have been brought to the classroom (and taken home again!) The lesson was about addition, not about weight. Our discussion in this Module about Weight does not distinguish between *weight* and *mass* because this is a difficult idea for Primary School pupils to understand. Essentially, mass is the amount of a body and weight is the force with which gravity pulls that body towards the ground. The Head Teacher may suggest that this lesson should be linked to Science.

A Head Teacher is also likely to be unhappy that only one calculation is done by a group or, effectively, one pupil in each group. Why isn't every pupil having work or activity rather than 90% of the class being passive on-lookers? More pupils need more work!

10.5 Time

"Think" answers

Throughout every hour, the hour hand on an analogue clock is rotating towards the next number.

"Watch" comment

In the same way that a number line supports addition, subtraction, multiplication and division of number work, a time line is very supportive for pupils learning how to calculate time intervals.

The video clip illustrates the teacher showing that the time line needs to record both the start time and the finish time in the same time recording system – either am/pm or 24-hour – recording midnight where there is a change of day.

"Reflect" answers

Very few local timetables are available, especially for transport that leaves when the vehicle is full rather than at a set time. Airline timetables have specific departure and arrival times but these are often only available on-line rather than as a printed notice. A calendar of events can be useful but these often only refer to days and so cannot be used for calculating with hours and minutes. Television programmes run according to a schedule and so can be used to calculate short periods of time involving hours and minutes.

Module 11:

11.1 Position and direction

"**Think**" answers

The Harmattan wind (and dust!) blows from the North to the South across West Africa.

"**Reflect**" answers

1. The video clip shows the teacher trying to establish an understanding of the cardinal points. A good continuation would be to use the turns and directions to follow a route on a map.

2. A child with learning difficulties may not necessarily be poor at the physical aspects of turning and following directions. He or she will need to be able to link the four cardinal directions with quarter turns of their body. Helping to establish which direction is East, towards the Kaaba, will be a best starting point.

   A visually-challenged pupil will also find it helpful to link the directions with quarter turns of the body.

"**Work with your partner**" comments

A sketch map of your local area will be more interesting for the class than a fictitious map which has no relevance for the children. A map of the State or of Nigeria will also be of interest and you can ask pupils to imagine themselves as the flight controller telling aircraft pilots which directions to turn to follow a route visiting some cities or avoiding some high mountains.

11.2 Lines and angles

"**Think**" answers

When you see an angle made by two lines, you naturally recognise the shape which the angle encloses. When you recognise that two angles are equal in size, it is probably the shape which your mind registers. So it is important that an introduction to angles emphasises the turning aspect which we do not "see".

“**Watch**” and “**Reflect**” answers

The three episodes show the teacher

- defining three types of angle;
- demonstrating these three types of angle with a pupil sitting and then asking pupils to make these angles with their arms; and playing the “Teacher says …” game to reinforce these notions;
- asking groups to use protractors to measure angles on an illustrated work-card.

The teacher demonstrated turning with the boy on the chair and turning was implied by the pupils moving their arms. However, the opportunity to relate the turning to the measuring of degrees with the protractor was not used. It does seem that the topic was not new to these pupils because the teacher did not need to show children how to use a protractor and so the notion of turning may have already been addressed in previous lessons. This final comment does pose the question of why the teacher chose this lesson plan as there was no obvious learning taking place. Perhaps this was a revision lesson for the Year 6 pupils?

“**Work with your partner**” comments

Using the movement of pointers/hands on an analogue clock face is an ideal way to link the angles with turning. For example, it is obvious that in moving through from 12 o/clock to about 12:16, to create a 90 degree angle, the hands have rotated.

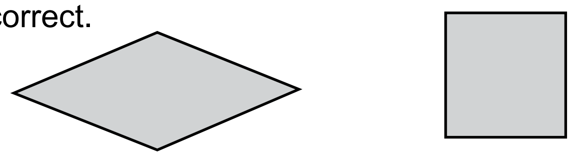
One corner of a piece of paper or book is 90 degrees, so holding the corner at the centre of a circular clock will identify many times when the two hands are at right angles. The illustration shows that the hands are at right angles at 13:25 (but not 17:08; *why not?*).

11.3 Plane shapes

“**Think**” answers

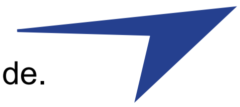
All five statements are correct.

“**Explore**” answers

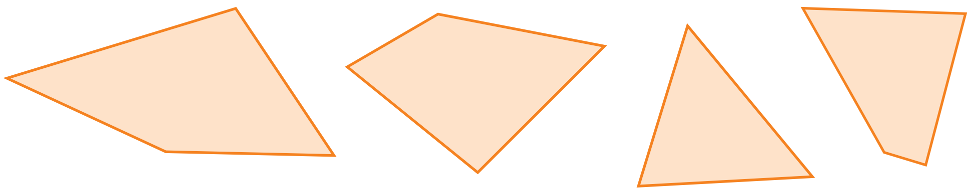


- An equilateral quadrilateral is a 4-sided shape with all sides the same length. This could be a rhombus or a square. (Note that a square is a rhombus with equal angles!)

- The green trapezium has no equal sides. The red trapezium has three equal sides.
- A parallelogram with four right-angles must be a rectangle (maybe, a square).



- This blue quadrilateral has a reflex angle inside.



- The trapezium above on the right has three equal sides, so do these four irregular quadrilaterals.

“**Reflect**” answers

The answer is “yes” to all four questions.

11.4 3-D shapes

“**Think**” answers

Matching shapes is a good homework task for pupils. It will help them to be observant and to recognise mathematical shapes in many different contexts. Finding close matching pairs is easier to do with 2-D flat shapes (such as a leaf and an ellipse) but children will recognise some close matching 3-D shapes too, including a flat-roofed house with a cuboid or a traditional Fulani hat with a cone.

“**Explore**” answers

There is a large variety of possible shapes to find when cutting across a bar of soap. If you do this as part of a classroom lesson, make sure you distinguish between 2-D shapes which describe the surfaces and 3-D shapes which describe the solids. (Choose a bar of soap that is not so dry that it crumbles, and not so soft that the edges and corners get squashed.)

"Reflect" comments

Sketching 3-D solids is a useful skill that your pupils may find difficult. You may like to spend a lesson in helping pupils to draw solid shapes. (Time is given to develop this skill in JSS.) Otherwise, to record information about 3-D solids in their exercise books, pupils will need the names of solids and the technical vocabulary to describe them. You will find guidance about this in the TDP Lesson Plans for Year 6 Week 26.

The example in the "**Work with your partner** …" section also suggests a way of avoiding drawing solids by using a table to describe the results of an investigation with cuboids.

A mobile phone can be used to capture photos of pupil's work to include in their exercise books.

11.5 Symmetry

"Think" answers

A line of symmetry is a dividing line which separates two equal but opposite halves. (It is one-dimensional, cutting a two-dimensional shape.)
A plane of symmetry is a slicing which separate two equal but opposite halves of a solid. (The plane is two-dimensional, cutting across a three-dimensional solid.)

"Reflect" answers

Teachers need to maintain a distinction between lines and planes of symmetry. A line of symmetry has no meaning for 3-D objects because a line cannot separate a solid into two equal halves.

Inevitably, cutting an object in half will spoil it, so your demonstrations need to cut shapes on paper (2-D) or solids such as fruit and vegetables that can be used later. Unfortunately, things like yams or plantain are rarely symmetrical enough to demonstrate the mathematical idea – an avocado pear is convenient or a banana cut lengthways will give the desired visual information to introduce the concept.

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